CHAPTER 1 – ESSENTIALS OF GEOMETRY

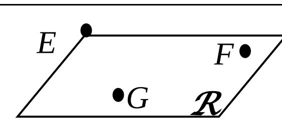
In this chapter we address three **Big IDEAS**:

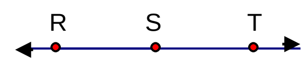
1) ***Describing geometric figures***
2) ***Measuring geometric figures***
3) ***Understanding equality and congruence***

Section:	**1 – 1 Identify Points, Lines, and Planes**
Essential Question	**How do you name geometric figures?**

Warm Up:

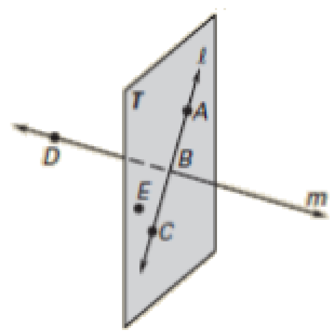
Key Vocab:

Undefined Terms		
A basic figure that is not defined in terms of other figures.		
Point	An undefined term in geometry Has no dimension – no length, width, or height. Designates a location	 "Point *A*"
Line	An undefined term in geometry Has one dimension – length A straight path that has no thickness and extends forever	 $\overleftrightarrow{ST}$; line *m*
Plane	An undefined term in geometry Has two dimensions – length and width A flat surface that has no thickness and extends forever in two dimensions	 Plane *EFG* or Plane *R*

Defined Terms		
Terms that can be described using other figures such as point or line		
Collinear Points	Points that lie on the same line.	
Coplanar Points	Points that lie in the same plane.	
Line Segment	Part of a line that consists of two points, called endpoints, and all points on the line that are between the endpoints.	 $\overline{BC}$
Ray	Half of a line that consists of one point called an endpoint and all points on the line that extend in one direction.	 $\overrightarrow{AB}$
Opposite Rays	Collinear rays, with a common endpoint, extending in opposite directions.	 $\overrightarrow{SR}$ and $\overrightarrow{ST}$ are opposite rays S is the common endpoint.
Intersection	The set of all points two or more figures have in common.	

Show:

Ex 1:



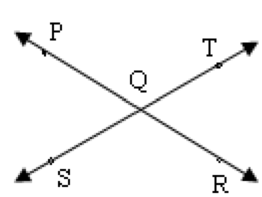
a. Give two other names for $\overleftrightarrow{BD}$.
   $\overleftrightarrow{DB}$ and m

b. Give another name for plane T.
   plane ABE, plane BEC, plane AEC

c. Name three points that are collinear.
   A, B, C

d. Name four points that are coplanar.
   A, B, C, E

Ex 2:



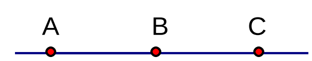
a. Give another name for $\overline{PR}$.
   $\overline{RP}$

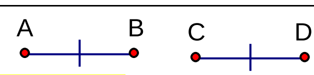
b. Name all rays with endpoint Q. Which of these rays are opposite rays?
   $\overrightarrow{QP}$, $\overrightarrow{QR}$, $\overrightarrow{QT}$, $\overrightarrow{QS}$;
   $\overrightarrow{QT}$ and $\overrightarrow{QS}$, $\overrightarrow{QP}$ and $\overrightarrow{QR}$ are opposite rays.

<table>
<tr><td>Section:</td><td>1 – 2 Use Segments and Congruence</td></tr>
<tr><td>Essential Question</td><td>What is the difference between congruence and equality?</td></tr>
</table>

Warm Up:

Key Vocab:

<table>
<tr><td>Postulate or Axiom</td><td colspan="2">A rule that is accepted without proof</td></tr>
<tr><td>Theorem</td><td colspan="2">A rule that can be proven</td></tr>
<tr><td>Between</td><td>When three points are collinear, you can say one point is between the other two.</td><td>

Point B is between points A & C</td></tr>
<tr><td rowspan="2">Congruent Segments</td><td colspan="2">Line segments that have the same length.</td></tr>
<tr><td colspan="2">

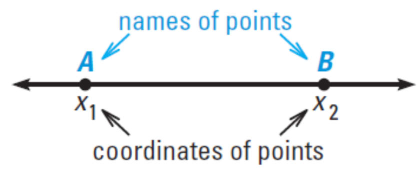
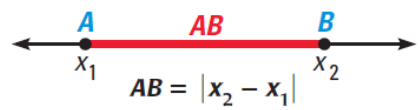
Lengths are equal | Segments are congruent
$AB = CD$ (is equal to) | $\overline{AB} \cong \overline{CD}$ (is congruent to)
a number = a number | A segment $\cong$ a segment</td></tr>
</table>

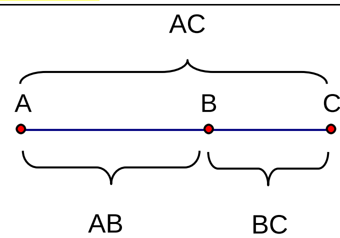
***It would be *incorrect* to say that two desks are equal. Do they have equal heights? Equal weights? Equal volumes? Height, weight, and volume all refer to numeric values that describe the desk. "Numbers are equal."**

***It would be *correct* to say that two desks are congruent. They have the same size and shape. "Objects are congruent."**

***Could two objects have the same height, but be differently shaped? Yes! Equality is not always a specific enough descriptor. This is the reason we use congruence.**

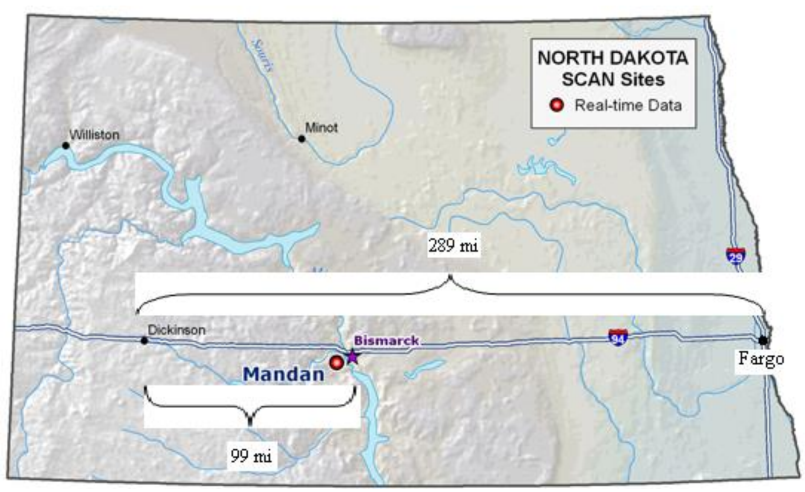
Postulates:

Ruler Postulate	
Allows for the creation of a measuring system.	
The points on a line can be matched one to one with the real numbers. The real number that corresponds to a point is the coordinate of the point.	
The distance between points *A* and *B*, written *AB*, is the absolute value of the difference of the coordinates of *A* and *B*.	

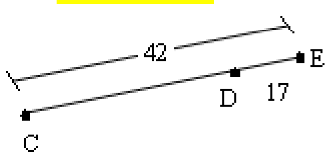
Segment Addition Postulate		
The sum of the parts equals the whole		
If *B* is between *A* and *C*,	**then** $AB + BC = AC$.	
If $AB + BC = AC$,	**then** *B* is between *A* and *C*.	

Show:

Ex 1: The cities shown on the map lie approximately in a straight line. Use the given distances to find the distance from Bismarck to Fargo. 190 mi



Ex 2: Find *CD*. 42 - 17 = 25



Ex 3: Point S is between R and T on $\overline{RT}$. Use the given information to write an equation in terms of *x*. Solve the equation. Then find *RS* and *ST*.

$RS = 3x - 16 \qquad ST = 4x - 8 \qquad RT = 60$

By the Segment Addition Postulate:

$$\begin{aligned} 3x - 16 + 4x - 8 &= 60 \\ 7x - 24 &= 60 \\ 7x &= 84 \\ x &= 12 \end{aligned}$$

$$RS = 3(12) - 16 = 20$$
$$ST = 4(12) - 8 = 40$$

Closure:

- Explain the difference between congruence and equality.

Congruence is used to describe *figures* that have the same size and shape.

Equality is used to describe the *size* of figures and refers to a number associated with that measurement.

Section:	**1 – 2 ½ Simplifying Radicals**
Essential Question	**How do you simplify radicals?**

Warm Up:

Key Vocab:

Square Root	If $r^2 = s$, then $\boldsymbol{r = \sqrt{s}}$ If the square of a number r is a number s, then **a number r** is **a square root of a number s** **Examples:** $2 = \sqrt{4}$ *two is the square root of four* $4 = \sqrt{16}$ *four is the square root of sixteen*
Radical	An expression of the form $\sqrt{s}$ or $\sqrt[n]{s}$ **Radical Sign** $\sqrt{32x^3}$ **Radicand**
Radicand	Number inside the radical sign
Simplest Radical Form	A radical expression is in simplest radical form if no radicand contains a factor (other than one) that is a perfect square AND every denominator has been rationalized **Non-Example:** $\sqrt{18}$ 9 *is perfect square factor of* 18. *Its simplest radical form is* $3\sqrt{2}$.
Rationalizing the Denominator	Rationalizing the denominator is a process of removing a radical from the denominator of a fraction. **Example:** $\frac{4}{\sqrt{3}}$ Step 1: $\frac{4}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$ Step 2: $\frac{4\sqrt{3}}{\sqrt{9}}$ Step 3: $\frac{4\sqrt{3}}{3}$

Key Concepts

Simplifying Radicals:	
$\left(\sqrt[n]{b}\right)^n = \sqrt[n]{b^n} = b$	A square root and a squared quantity cancel out
$\sqrt{ab} = \sqrt{a} \bullet \sqrt{b}$	**The square root of a product** is **the product of the square roots** → You can break apart multiplication
$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$	**The square root of a quotient** is **the quotient of the square roots** → You can break apart division
➢ $\sqrt{a^2+b^2} \neq \sqrt{a^2} + \sqrt{b^2}$	**Caution!** The square root of a sum is ***NOT*** the sum of the square roots →You CANNOT break apart addition!

Show:

Simplify.

1. $\sqrt{50} = 5\sqrt{2}$

2. $\sqrt{56} = 2\sqrt{14}$

3. $\sqrt{12} = 2\sqrt{3}$

4. $\sqrt{10}$ Simplified

5. $\sqrt{5^2} = 5$

6. $\sqrt{x^2} = x$

7. $\sqrt{(-3)^2} = 3$

8. $\sqrt{(-t)^2} = t$

9. $\sqrt{25 \bullet 9} = \sqrt{25} \bullet \sqrt{9} = 5 \bullet 3 = 15$

10. $\sqrt{x^2 y^2} = xy$

11. $\sqrt{\frac{16}{25}} = \frac{4}{5}$

12. $\sqrt{\frac{r^2}{w^2}} = \frac{r}{w}$

13. $\sqrt{\frac{1}{2}} \bullet \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

14. $\sqrt{\frac{2}{5}} \bullet \frac{\sqrt{5}}{\sqrt{5}} = \frac{\sqrt{10}}{5}$

15. $\sqrt{\left(\frac{a}{2}\right)^2} = \frac{a}{2}$

16. $\sqrt{\left(\frac{-a}{2}\right)^2} = \frac{a}{2}$

17. $\sqrt{9+25} = \sqrt{34}$

18. $\sqrt{g^2 + h^2}$ Simplified

19. $\sqrt{16(s+t)} = 4\sqrt{s+t}$

20. $\sqrt{4n^2 + 4m^2} = \sqrt{4\left(n^2 + m^2\right)}$

$= 2\sqrt{n^2 + m^2}$

Closure:

- How can you use decimals to check your answers?

Compare the decimal approximation of the original problem to the decimal approximation of your answer. If they match, you have simplified correctly. You simply need to make sure that you simplified COMPLETELY.

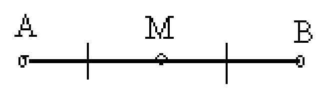
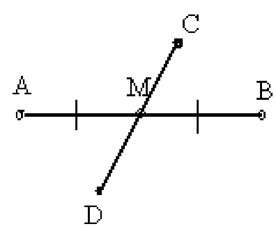
- How do you know when a square root is fully simplified?

A square root is simplified when there are no perfect square factors inside the radical AND there are no radicals in the denominator.

Section:	**1 – 3 Use Midpoint and Distance Formulas**
Essential Question	**How do you find the distance and the midpoint between two points in the coordinate plane?**

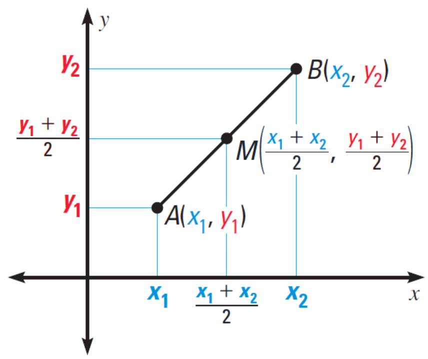
Warm Up:

Key Vocab:

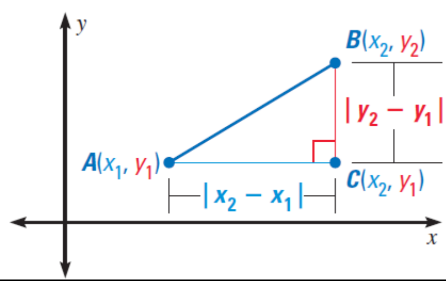
Midpoint	The point that divides the segment into two congruent segments.	 M is the midpoint of $\overline{AB}$
Segment Bisector	A point, ray, line, line segment, or plane that intersects the segment at its midpoint.	 $\overline{CD}$ is a segment bisector of $\overline{AB}$

Key Concepts:

Midpoint Formula	
If $A(x_1, y_1)$ and $B(x_2, y_2)$ are points on a coordinate plane,	**then** the midpoint M of $\overline{AB}$ has coordinates $\left(\dfrac{x_1+x_2}{2}, \dfrac{y_1+y_2}{2}\right)$

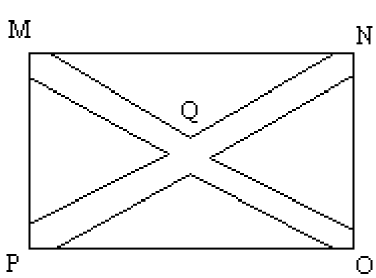


Distance Formula	
If $A(x_1, y_1)$ and $B(x_2, y_2)$ are points in a coordinate plane,	**then** the distance between A and B is $AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

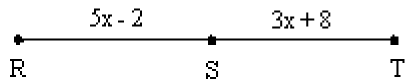


Show:

Ex 1: The figure shows a gate with diagonal braces. $\overline{MO}$ bisects $\overline{NP}$ at Q. If PQ=22.6 in., find PN. By the definition of a segment bisector PN = 45.2 in



Ex 2: Point S is the midpoint of $\overline{RT}$. Find ST.



$$5x-2=3x+8$$
$$2x=10$$
$$x=5$$

$$ST=3(5)+8=23$$

Ex 3: Find PQ given the coordinates for its endpoints are $P(2,5)$ and $Q(-4,8)$. Give an exact answer AND approximate answer rounded to the nearest hundredth.

$$\begin{aligned} PQ &= \sqrt{(-4-2)^2+(8-5)^2} \\ &= \sqrt{36+9} \\ &= \sqrt{45} \\ &= 3\sqrt{5} \approx 6.71 \end{aligned}$$

Exact Answer　　　　　　　　Approximate Answer

Ex 4:

a. The endpoints of $\overline{GH}$ are G(7, -2) and H(-5, -6). Find the coordinates of the midpoint P.

$$\left(\frac{7-5}{2},\frac{-2--6}{2}\right)=(1,-4)$$

b. The midpoint of $\overline{AB}$ is M(5, 8). One endpoint is A(2, -3). Find the coordinates of the other endpoint B.

x:

$$5=\frac{2+x}{2}$$
$$10=2+x$$
$$8=x$$

y:

$$8=\frac{-3+y}{2}$$
$$16=-3+y$$
$$19=y$$

B(8, 19)

c. The midpoint of $\overline{GH}$ is M(4, -1). One endpoint is G(5, 3). Find the coordinates of the other endpoint H.

x:

$$4=\frac{5+x}{2}$$
$$8=5+x$$
$$3=x$$

y:

$$-1=\frac{3+y}{2}$$
$$-2=3+y$$
$$-5=y$$

H(3, -5)

Closure:

- Write the midpoint and distance formulas.

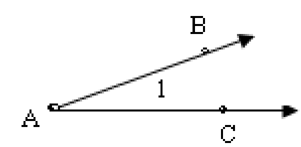
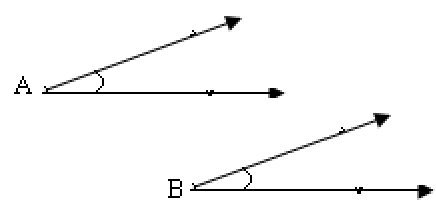
Distance formula: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

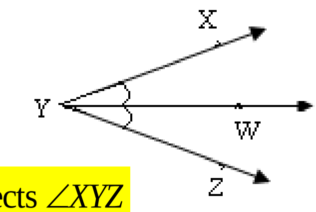
Midpoint formula: $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$

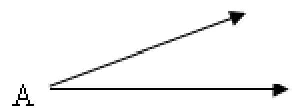
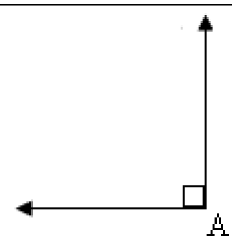
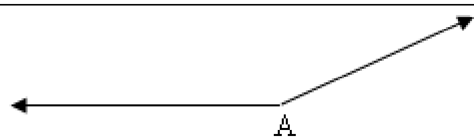
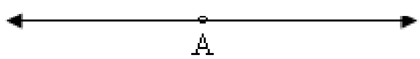
Section:	**1 – 4 Measure and Classify Angles**
Essential Question	**How do you identify whether an angle is acute, right, obtuse, or straight?**

Warm Up:

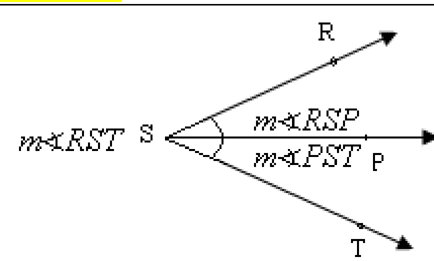
Key Vocab:

Angle	Two different rays with the same endpoint **Notation:** □ *BAC*, □ *CAB*, □ *A*, □ 1 $\angle BAC,\ \angle CAB,\ \angle A,\ \angle 1$	
Sides	The rays are the sides of the angle **Notation:** $\overrightarrow{AB},\ \overrightarrow{AC}$	
Vertex	The common endpoint of the rays	
Congruent Angles	Angles that have the same measure	 $\angle A \cong \angle B$ and $m\angle A = m\angle B$

Angle Bisector	A ray that divides an angle into two congruent angles. ***segment bisector $\neq$ angle bisector***	 $\overrightarrow{YW}$ bisects $\angle XYZ$ $\therefore \angle XYW \cong \angle WYZ$

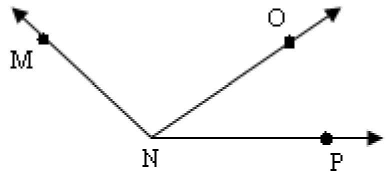
Classifying Angles		
Acute Angle		$0° < m\angle A < 90°$
Right Angle		$m\angle A = 90°$
Obtuse Angle		$90° < m\angle A < 180°$
Straight Angle		$m\angle A = 180°$

Postulate:

Angle Addition Postulate		
The sum of the parts equals the whole		
If P is in the interior of $\angle RST$,	**Then** $m\angle RST = m\angle RSP + m\angle PST$	

Show:

Ex 1: Name each angle that has N as a vertex. $\angle MNO,\ \angle ONP,\ \angle MNP$



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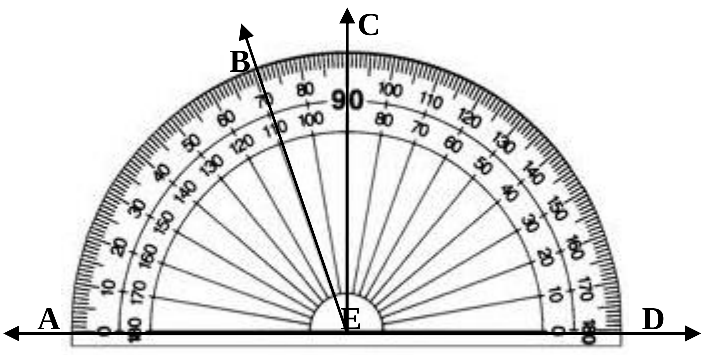
Ex 2: Use the diagram to find the measure of each angle and classify the angle.



a. $\angle DEC$ ___90° Right______

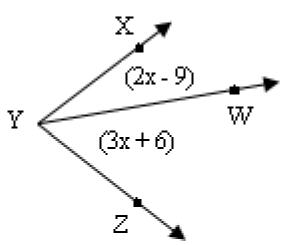
b. $\angle DEA$ ___180° Straight__

c. $\angle CEB$ ____20° Acute____

d. $\angle DEB$ ____110° Obtuse__

Ex 3: If $m\angle XYZ = 72°$, find $m\angle XYW$ and $m\angle ZYW$.

By the Angle Addition Postulate:

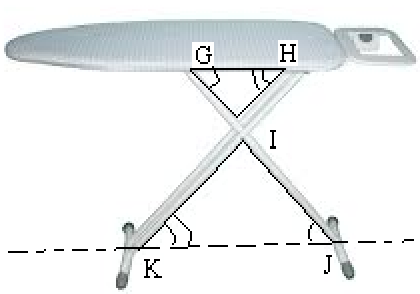


$$\begin{aligned} 2x - 9 + 3x + 6 &= 72 \\ 5x - 3 &= 72 \\ 5x &= 75 \\ x &= 15 \end{aligned}$$

$m\angle XYW = 2(15) - 9 = 21°$

$m\angle ZYW = 3(15) + 6 = 51°$

Ex 4: The figure shows angles formed by the legs of an ironing board. Identify the congruent angles. If $m\angle HGI = 40°$, what is $m\angle GJK$?



$\angle HGI \cong \angle GJK,\ \angle GHK \cong \angle HKJ;$
$m\angle GJK = 40°$

Closure:

- Explain the difference between congruence and equality in terms of angles.

Angles are said to be congruent while their *measurements* are said to be equal, for example $40° = 40°$.

- What are the ways to classify angles?

Acute angles have measure between 0 degrees and 90 degrees.
Right angles have measure of exactly 90 degrees.
Obtuse angles have measures between 90 degrees and 180 degrees.
Straight angles have measure of exactly 180 degrees.

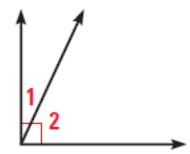
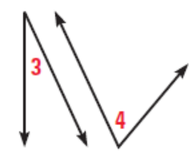
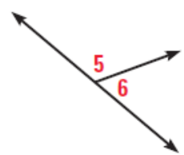
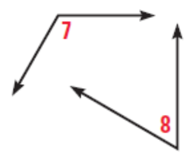
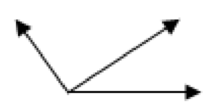
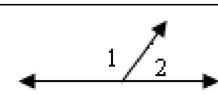
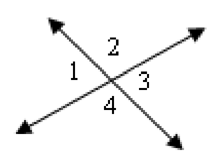
- Sketch a diagram: $\overrightarrow{LM}$ bisects $\angle ALB$

- Sketch a diagram: $\overrightarrow{LM}$ bisects $\overline{AB}$

Section:	**1 – 5 Describe Angle Pair Relationships**
Essential Question	How do you identify complementary and supplementary angles?

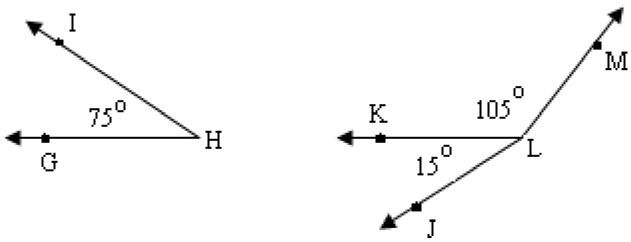
Warm Up:

Key Vocab:

Complementary Angles	Two angles whose sum is $90°$	 Adjacent  Non-adjacent
Supplementary Angles	Two angles whose sum is $180°$	 Adjacent  Non-adjacent
Adjacent Angles	Two angles that share a common vertex and side, but have no common interior points	
Linear Pair	Two adjacent angles whose noncommon sides are opposite rays	
Vertical Angles	Two angles whose sides form two pairs of opposite rays **Examples:** $\angle 1$ and $\angle 3$ $\angle 2$ and $\angle 4$	

Show:

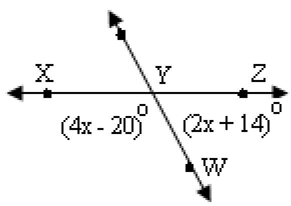
Ex 1: In the figure, name a pair of complementary angles, a pair of supplementary angles, and a pair of adjacent angles. $\angle GHI, \angle JLK; \angle GHI, \angle KLM; \angle JLK, \angle KLM$



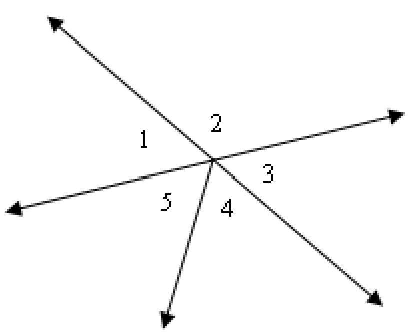
Ex 2: a. Given that $\angle 1$ is a complement of $\angle 2$ and $m\angle 1 = 17°$, find $m\angle 2$. 73°

b. Given that $\angle 3$ is a supplement of $\angle 4$ and $m\angle 3 = 119°$, find $m\angle 4$. 61°

Ex 3: Two roads intersect to form supplementary angles, $\angle XYW$ and $\angle WYZ$. Find $m\angle XYW$ and $m\angle WYZ$. 76°, 104°



Ex 4: Identify all of the linear pairs and all of the vertical angles in the figure.



Linear pairs: $\angle 2$ and $\angle 3$; $\angle 1$ and $\angle 3$

Vertical angles: $\angle 1$ and $\angle 3$

Ex 5: Two angles form a linear pair. The measure of one angle is 3 times the measure of the other angle. Find the measure of each angle.

$$\begin{aligned} x + y &= 180 \qquad & x &= 3y \\ 3y + y &= 180 & x &= 3(45) \\ 4y &= 180 & x &= 135^\circ \\ y &= 45^\circ \end{aligned}$$

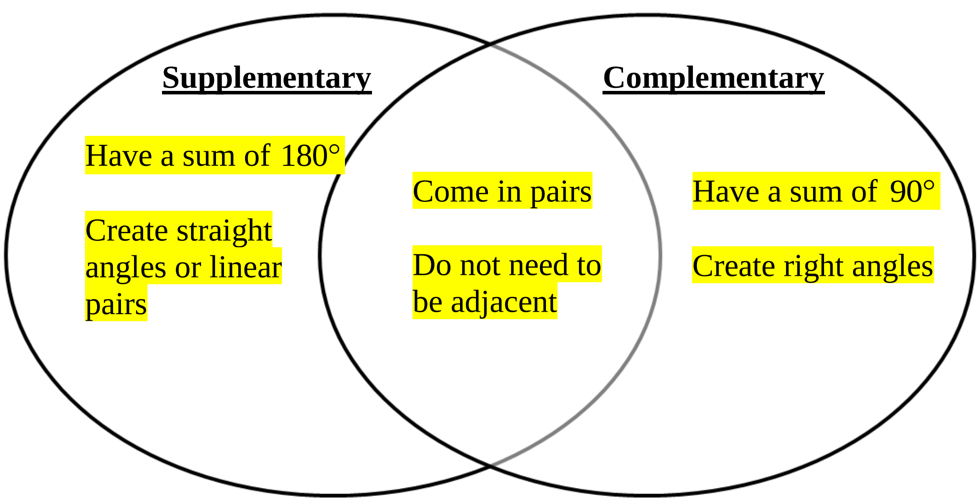
Ex 6: The measure of one angle is 7 times the measure of its complement. Find the measure of each angle.

$$\begin{aligned} x &= 7(90 - x) \\ 8x &= 630 \\ x &= 78.75^\circ \end{aligned}$$

$$90 - 78.75 = 11.75^\circ$$

Closure:

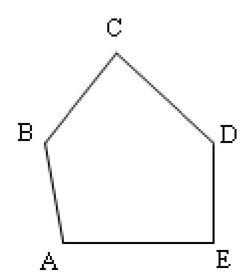
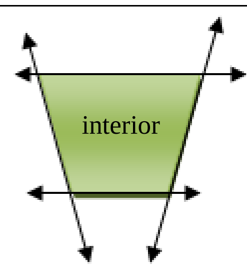
- Compare and contrast complementary and supplementary angles.

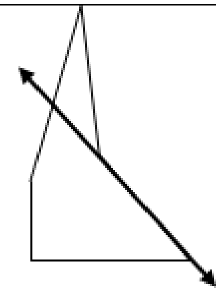
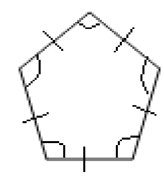


Section:	**1 – 6 Classify Polygons**
Essential Question	How do you classify polygons?

Warm Up:

Key Vocab:

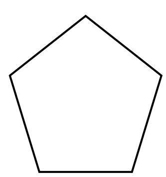
Polygon	A closed plane figure with three or more sides each side intersects exactly two sides, one at each endpoint, so that no two sides with a common endpoint are collinear	 Sides: $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, $\overline{DE}$, and $\overline{AE}$ Vertices: *A*, *B*, *C*, *D* and *E*
Sides	Each line segment that forms a polygon	
Vertex	Each endpoint of a side of a polygon	
Convex	A polygon where no line containing a side of the polygon contains a point in the interior of the polygon All interior angles measures are less than 180°	

Concave	A polygon with one or more interior angles measuring greater than 180° Opposite of convex	
n-gon	A polygon with *n* sides	**Example:** A polygon with 14 sides is a 14-gon
Equilateral	A polygon with all of its sides congruent	
Equiangular	A polygon with all of its interior angles congruent	
Regular	A convex polygon that has all sides and all angles congruent	Regular Pentagon

Show:

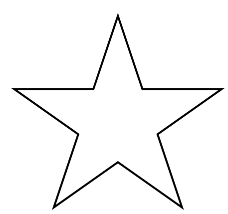
Ex 1: Tell whether each figure is a polygon. If it is, tell whether it is concave or convex.

a.



Yes; Convex

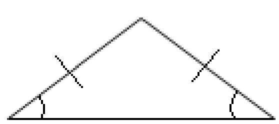
b.



Yes; Concave

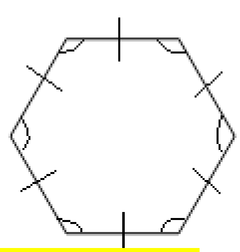
Ex 2: Classify the polygon by the number of sides. Tell whether the polygon is equilateral, equiangular, or regular. Explain your reasoning.

a.



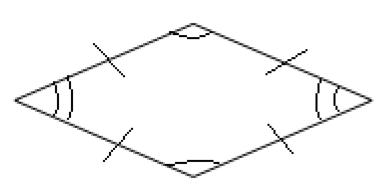
Triangle; only 2 sides congruent, only 2 angles congruent, so not equilateral, not equiangular, not regular.

b.



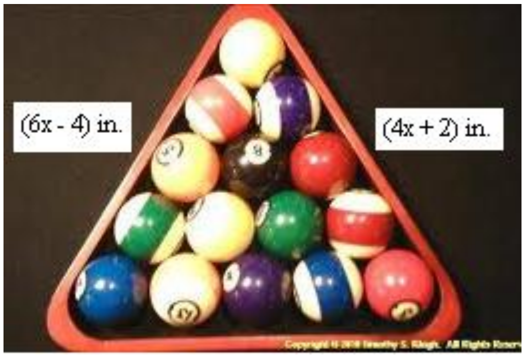
Hexagon; equilateral, equiangular, regular

c.



Quadrilateral; equilateral, not equiangular, so not regular

Ex 3: A rack for billiard balls is shaped like an equilateral triangle. Find the length of a side.



$$6x - 4 = 4x + 2$$
$$2x = 6$$
$$x = 3$$

$$6(3) - 4 = 4(3) + 2 = 14 \text{ in}$$