CHAPTER 8 – QUADRILATERALS

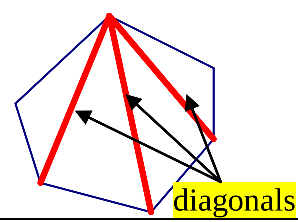
In this chapter we address three **Big IDEAS**:

1) ***Using angle relationships in polygons.***
2) ***Using properties of parallelograms.***
3) ***Classifying quadrilaterals by the properties.***

Section:	**8 – 1 Find Angle Measures in Polygons**
Essential Question	**How do you find a missing angle measure in a convex polygon?**

Warm Up:

Key Vocab:

Diagonal	A segment that joins two *nonconsecutive* vertices of a polygon.	

Theorems:

Polynomial Interior Angles Theorem: The sum of the measures of a convex *n*-gon is $(n-2)\cdot 180°$

Corollary - Interior Angles of a Quadrilateral: The sum of the measures of the interior angles of a quadrilateral is 360^0.

Polynomial Exterior Angles Theorem: The sum of the measures of the exterior angles of a convex polygon, one angle at each vertex, is 360^0.

Area of a Regular Polygon: $A=\frac{1}{2}ap$ where *a* is the *apothem* and *p* is the perimeter.

Show:

Ex 1: Find the sum of the measures of the interior angles of a convex decagon.

$(10-2)180 = 1440°$

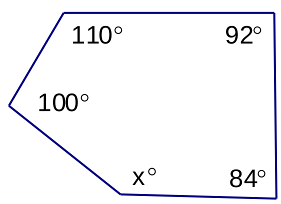
Ex 2: The sum of the measures of the interior angles of a convex polygon is 2340^0. Classify the polygon by the number of sides.

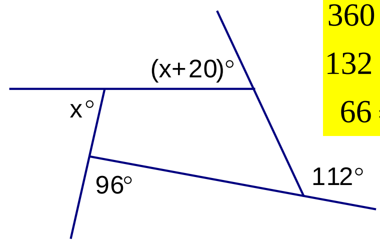
$$2340 = (n-2)180$$
$$13 = n-2$$
$$15 = n$$

The polygon is an 15-gon

Ex 3: Find the value of x in each of the diagrams shown below.

$$(5-2)180 = 540$$
$$x = 540 - 110 - 100 - 92 - 84$$
$$x = 154$$





$$360 = x + x + 20 + 96 + 112$$
$$132 = 2x$$
$$66 = x$$

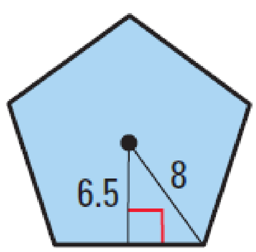
Ex 5: Find the area of each regular polygon.

a)

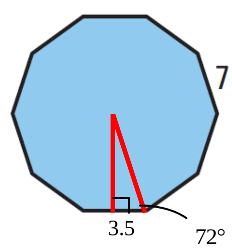
$$6.5^2 + b^2 = 8^2$$
$$b = \sqrt{64 - 42.25}$$
$$b = \sqrt{21.75}$$

$p = 10b \approx 46.6$

$$A = \frac{1}{2}(6.5)(46.6)$$
$$A \approx 151.45$$



b)



$P = 7(10) = 70$

Interior angle measure:
$$\frac{(10-2)180}{10} = 144°$$
$$144 \div 2 = 72°$$

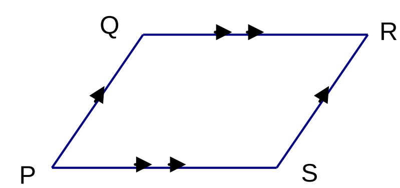
$$\tan 72 = \frac{a}{3.5}$$
$$3.5 \tan 72 = a$$

$$A = \frac{1}{2}(3.5 \tan 72)(70)$$
$$A \approx 377.0$$

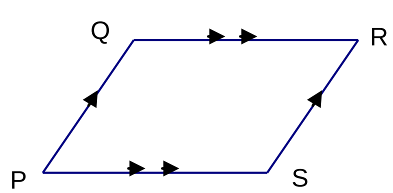
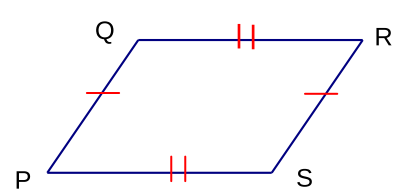
Section:	**8 – 2 Use Properties of Parallelograms**
Essential Question	How do you find angle and side measures in a parallelogram?

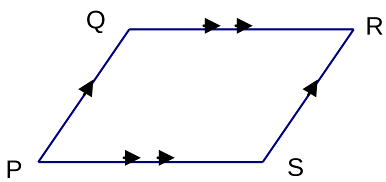
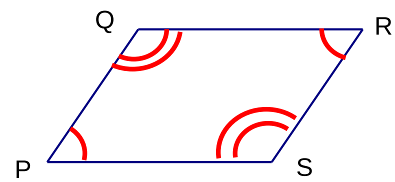
Warm Up:

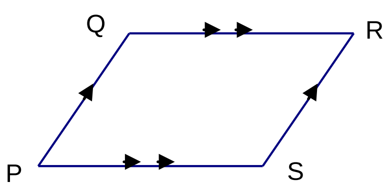
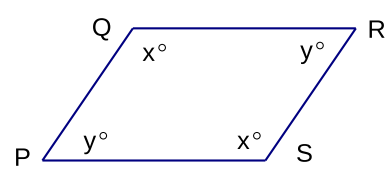
Key Vocab:

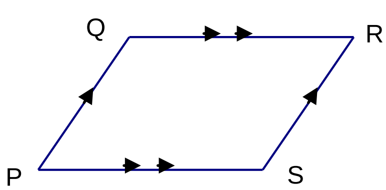
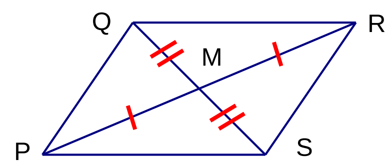
Parallelogram	A quadrilateral with BOTH pairs of opposite sides parallel.	 $\square$ *PQRS*

Theorems:

If	Then
a quadrilateral is a parallelogram,	its opposite sides are congruent.
$\square PQRS$	$\overline{PQ} \cong \overline{RS}$ and $\overline{QR} \cong \overline{PS}$
	

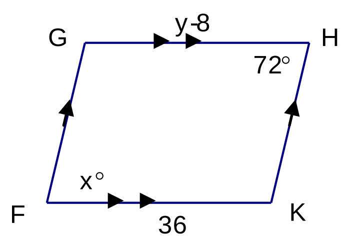
If	**Then**
a quadrilateral is a parallelogram,	its opposite angles are congruent.
▱ $PQRS$	$\angle P \cong \angle R$ and $\angle Q \cong \angle S$.
	

If	**Then**
a quadrilateral is a parallelogram,	its consecutive pairs of angles are supplementary.
▱ $PQRS$	$x° + y° = 180°$.
	

If	**Then**
a quadrilateral is a parallelogram,	its diagonals bisect each other.
▱ $PQRS$	$\overline{QM} \cong \overline{MS}$ and $\overline{PM} \cong \overline{RM}$.
	

Show:

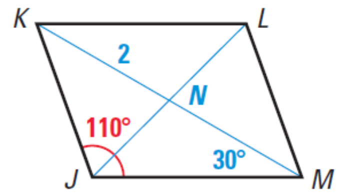
Ex1: Find the values of x and y.



$x° = 72°$

$y - 8 = 36$

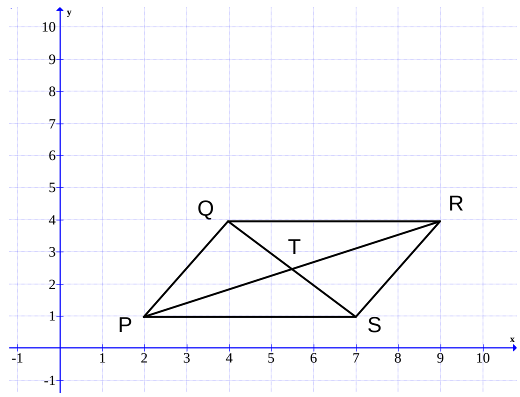
Ex2: Find the indicated measure.



a) $NM = 2$ **b)** $KM = 4$

c) $m\angle JML = 70°$ **d)** $m\angle KML = 40°$

Ex3: The diagonals of parallelogram PQRS intersect at point T. What are the coordinates of point T?



A. $\left(\frac{9}{2}, \frac{5}{2}\right)$

B. $\left(\frac{9}{2}, \frac{7}{2}\right)$

C. $\left(\frac{11}{2}, \frac{5}{2}\right)$

D. $\left(\frac{11}{2}, \frac{7}{2}\right)$

The diagonals of a parallelogram bisect each other, so T is the midpoint of $\overline{PR}$

$$T:\left(\frac{2+9}{2}, \frac{1+4}{2}\right) = \left(\frac{11}{2}, \frac{5}{2}\right)$$

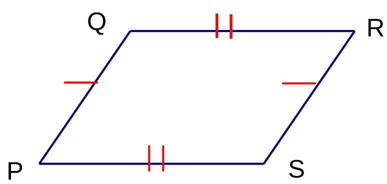
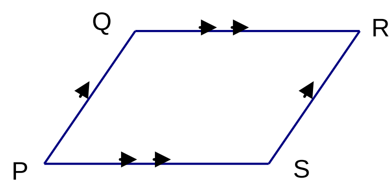
Closure:

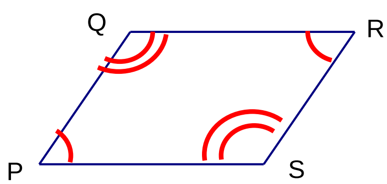
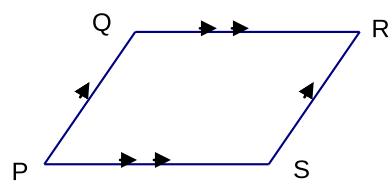
- What are the properties of a parallelogram?
  A parallelogram's opposite sides are parallel and congruent. Its opposite pairs of angles are congruent. Its consecutive pairs of angles are supplementary. Its diagonals bisect each.

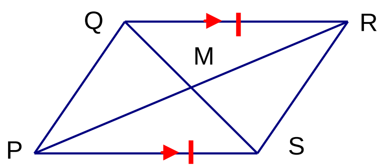
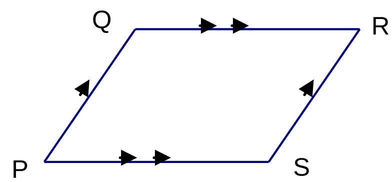
Section:	**8 – 3 Show that a Quadrilateral is a Parallelogram**
Essential Question	How can you prove that a quadrilateral is a parallelogram?

Warm Up:

Theorems:

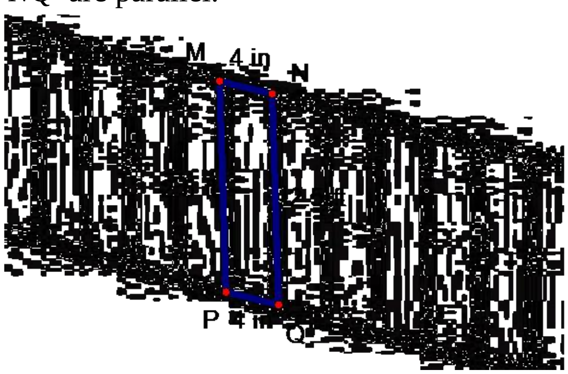
If	**Then**
both pairs of opposite sides of a quadrilateral are congruent,	the quadrilateral is a parallelogram.
$\overline{PQ} \cong \overline{RS}$ and $\overline{QR} \cong \overline{PS}$	▱ $PQRS$
	

If	**Then**
both pairs of opposite angles of a quadrilateral are congruent,	the quadrilateral is a parallelogram.
$\angle P \cong \angle R$ and $\angle Q \cong \angle S$.	▱ $PQRS$
	

If	**Then**
one pair of opposite sides of a quadrilateral is congruent AND parallel,	the quadrilateral is a parallelogram.
$\overline{QR} \cong \overline{PS}$ and $\overline{QR} \parallel \overline{PS}$ OR $\overline{PQ} \cong \overline{RS}$ and $\overline{PQ} \parallel \overline{RS}$,	$\square PQRS$
	

Show:

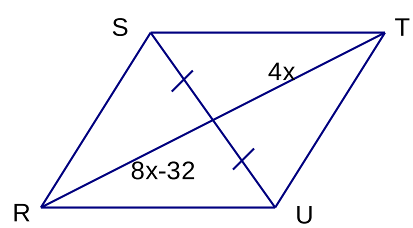
Ex1: The figure shows part of a stair railing. Explain how you know the support bars $\overline{MP}$ and $\overline{NQ}$ are parallel.



Since $MP = NQ$ and $MN = PQ$, $MNQP$ is a parallelogram.
Therefore, $\overline{MP} \parallel \overline{NQ}$

Ex2: For what value of x is quadrilateral $RSTU$ a parallelogram?

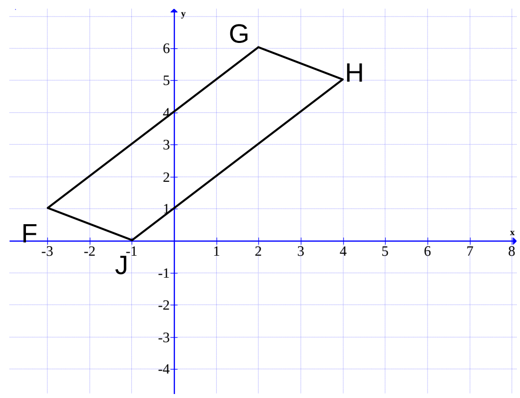
$$8x - 32 = 4x$$
$$4x = 32$$
$$x = 8$$



Ex3: Suppose you place two straight narrow strips of paper of equal length on top of two lines of a sheet of notebook paper. If you draw a segment to join their left ends and a segment to join their right ends, will the resulting figure be a parallelogram? Explain.

Yes, since the segments are congruent AND the lines on the notebook paper are parallel, we can use the theorem that says "If one pair of opposite sides of a quadrilateral is congruent and parallel, then the quadrilateral is a parallelogram"

Ex4: Show that $FGHJ$ is a parallelogram.



Option 1: Show BOTH pair of opposite sides congruent.

Option 2: Show one pair of opposite sides congruent AND parallel (have the same slope.)

Option 3: Show BOTH pair of opposite sides parallel.

For example: $FJ = GH = \sqrt{5}$

$$m_{\overline{FJ}} = m_{\overline{GH}} = \frac{-1}{2}$$

$$\therefore \square FGHJ$$

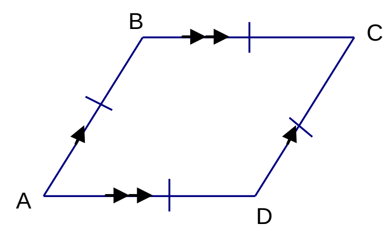
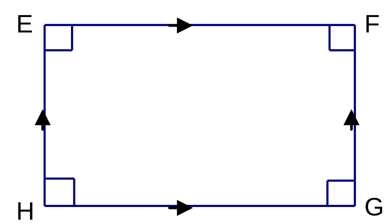
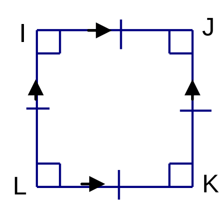
Closure:

- How do you prove that a quadrilateral is a parallelogram?
  Show that the quadrilateral has…
  1. both pair of opposite sides parallel.
  2. both pair of opposite sides congruent.
  3. one pair of opposites sides parallel AND congruent.
  4. both pair of opposite angles congruent.
  5. diagonals that bisect each other.

Section:	**8 – 4 Properties of Rhombuses, Rectangles, and Squares**
Essential Question	What are the properties of parallelograms that have all sides or all angles congruent?

Warm Up:

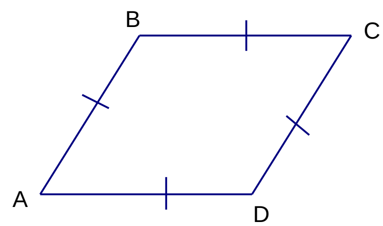
Key Vocab:

Rhombus	A parallelogram with four congruent sides	 $\overline{AB} \cong \overline{BC} \cong \overline{CD} \cong \overline{AD}$
Rectangle	A parallelogram with four right angles	 $m\angle E = m\angle F = m\angle g = m\angle H = 90^\circ$
Square	A parallelogram with four congruent sides AND four right angles	 $\overline{IJ} \cong \overline{JK} \cong \overline{KL} \cong \overline{LI}$

Theorems:

Rhombus Corollary

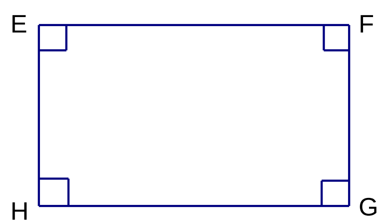
A quadrilateral is a rhombus IFF it has four congruent sides.



If	Then
If $\overline{AB} \cong \overline{BC} \cong \overline{CD} \cong \overline{AD}$,	Then quad $ABCD$ is a rhombus.
If quad $ABCD$ is a rhombus,	Then $\overline{AB} \cong \overline{BC} \cong \overline{CD} \cong \overline{AD}$.

Rectangle Corollary

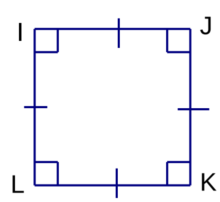
A quadrilateral is a rectangle IFF if it has four right angles.



If	Then
If $m\angle E = m\angle F = m\angle g = m\angle H = 90°$,	Then quad $ABCD$ is a rectangle.
If quad$ABCD$ is a rectangle,	Then $m\angle E = m\angle F = m\angle g = m\angle H = 90°$.

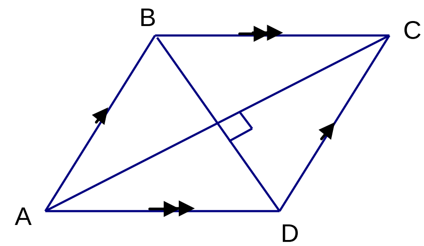
Square Corollary

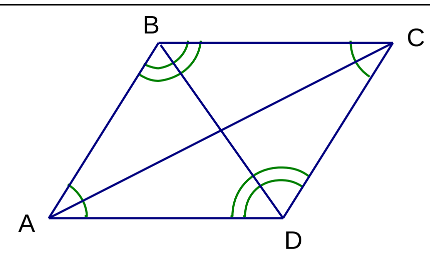
A quadrilateral is a square IFF it is a rhombus AND a rectangle.

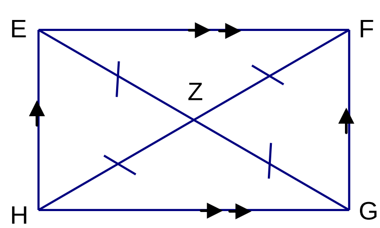


If	Then
If $\overline{IJ} \cong \overline{JK} \cong \overline{KL} \cong \overline{LI}$ AND $m\angle I = m\angle J = m\angle K = m\angle L = 90°$,	Then quad $ABCD$ is a square.
If quad $ABCD$ is a rectangle,	Then $\overline{IJ} \cong \overline{JK} \cong \overline{KL} \cong \overline{LI}$ AND $m\angle I = m\angle J = m\angle K = m\angle L = 90°$.

Theorems:

A parallelogram is a rhombus IFF its diagonals are perpendicular.	
	
If $\overline{AC} \perp \overline{BD}$,	**Then** $\square ABCD$ is a rhombus.
If $\square ABCD$ is a rhombus,	**Then** $\overline{AC} \perp \overline{BD}$.

A parallelogram is a rhombus IFF each diagonal bisects a pair of opposite angles.	
	
If $\overline{BD}$ bisects $\angle B$ AND $\angle D$ and $\overline{AC}$ bisects $\angle A$ AND $\angle C$,	**Then** $\square ABCD$ is a rhombus.
If $\square ABCD$ is a rhombus,	**Then** $\overline{BD}$ bisects $\angle B$ AND $\angle D$ and $\overline{AC}$ bisects $\angle A$ AND $\angle C$.

A parallelogram is a rectangle IFF its diagonals are congruent.	
	
If $\overline{EG} \cong \overline{HF}$,	**Then** $\square EFGH$ is a rectangle.
If $\square EFGH$ is a rectangle,	**Then** $\overline{EG} \cong \overline{HF}$.

Show:

Ex1: For any rectangle *ABCD*, decide whether the statement is *always*, *sometimes* or *never* true.

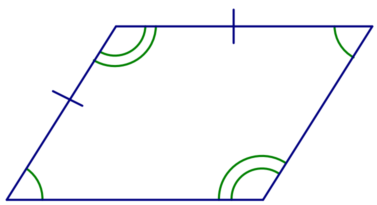
a.) $\overline{AB} \cong \overline{CD}$

Always; All rectangle are parallelograms and opposite sides of a parallelogram are congruent.

b.) $\overline{AB} \cong \overline{BC}$

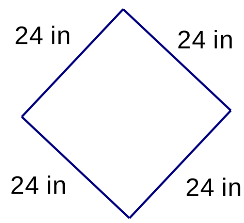
Sometimes; $\overline{AB} \cong \overline{BC}$ provided that the rectangle *ABCD* is a square. But not all rectangles are squares.

Ex2: Classify the special quadrilateral. Explain your reasoning.



It is a rhombus. Its is a parallelogram because opposite angles are congruent. Since a pair of adjacent sides are congruent, all four side are congruent.

Ex3: You are building a case with glass shelves for collectibles.



a.) Given the shelf measurements in the diagram, can you assume that the shelf is a square? Explain.

No, it has four congruent sides so it is a rhombus. However, we do not know whether the angles are right angles.

b.) You measure the diagonals and find they are both 33.94 inches. What can you conclude about the shape?

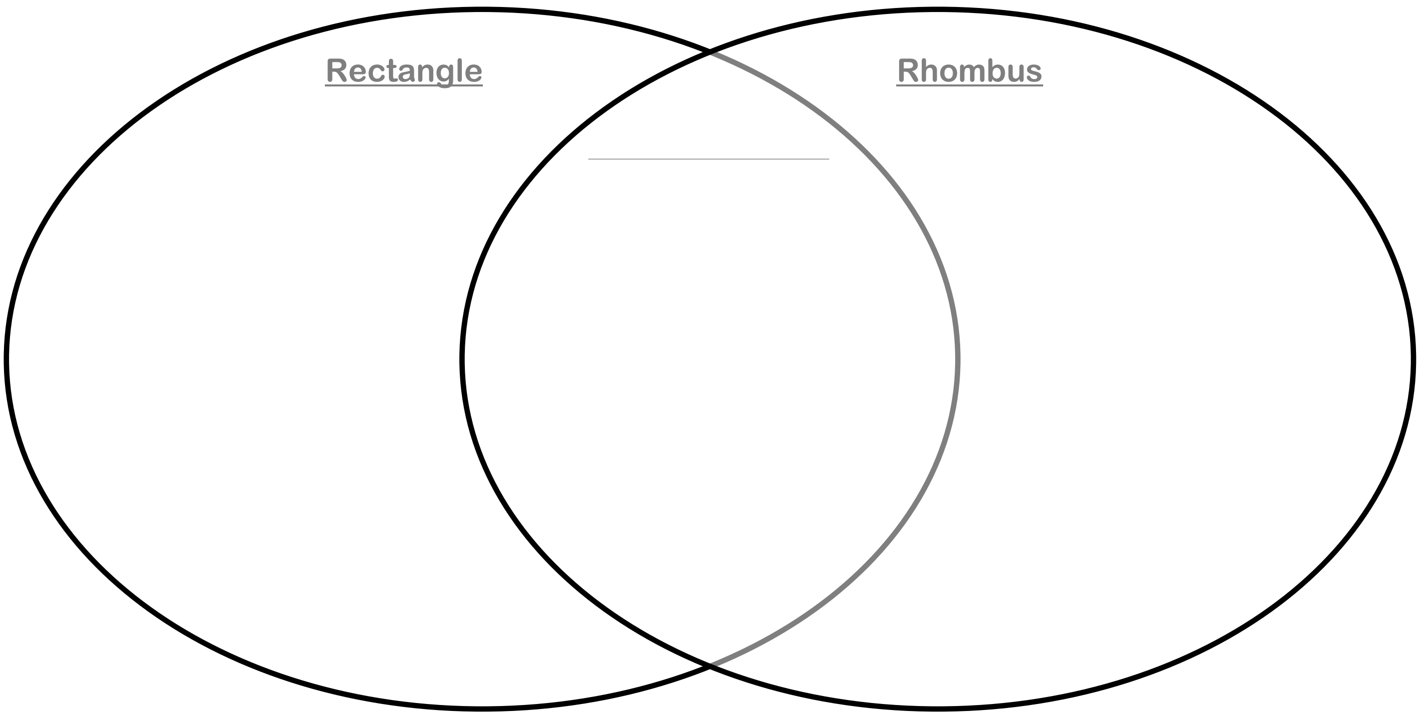
It is a square.

Ex4: Sketch a square *EFGH*. List everything that you know about it.

- Opposite sides are parallel
- All sides are congruent
- All angles are congruent right angles
- The diagonals are congruent and perpendicular and they bisect each other
- Each diagonal bisects a pair of opposite angles.

Closure:

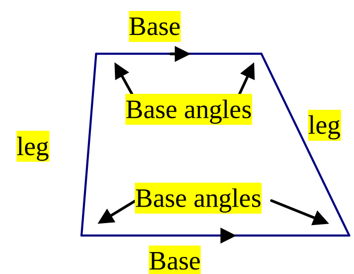
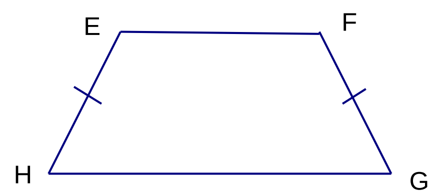
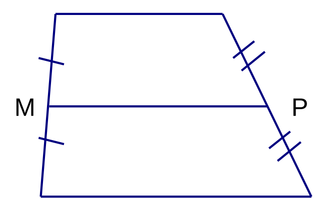
- Complete the Venn diagram for the properties that are ALWAYS true.

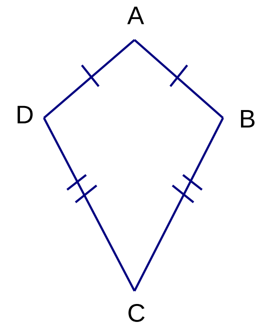


Section:	**8 – 5 Use Properties of Trapezoids and Kites**
Essential Question	What are the main properties of trapezoids and kites?

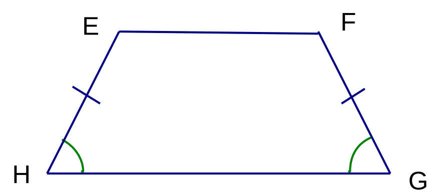
Warm Up:

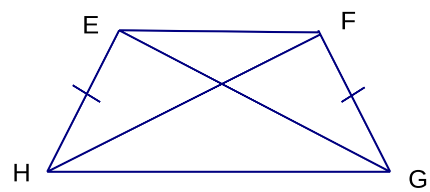
Key Vocab:

Trapezoid	A quadrilateral with *exactly* one pair of parallel sides.	
Bases	The parallel sides of a trapezoid.	
Base Angles	Either pair of angles whose common side is a base of a trapezoid.	
Legs	The nonparallel sides of a trapezoid.	
Isosceles Trapezoid	A trapezoid with congruent legs.	 $\overline{EH} \cong \overline{FG}$
Midsegment of a Trapezoid	A segment that connects the midpoints of the legs of a trapezoid.	 $\overline{MP}$ is the midsegment

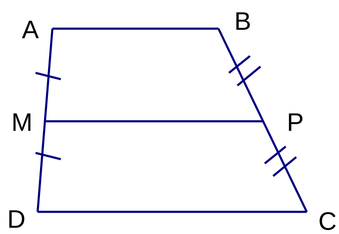
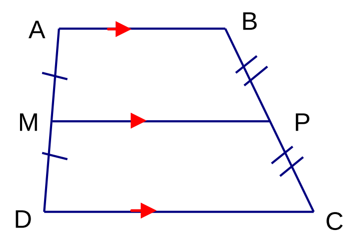
Kite	A quadrilateral that has two pairs of *consecutive congruent sides*, but in which *opposite sides* are NOT congruent.	 $\overline{AD} \cong \overline{AB}$ and $\overline{CD} \cong \overline{BC}$

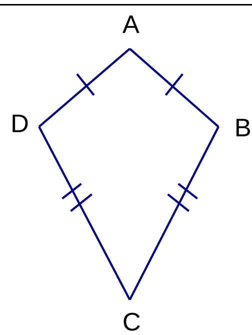
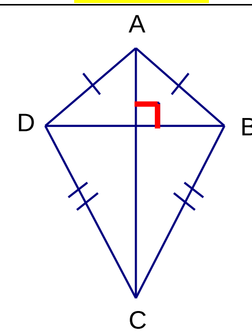
Theorems:

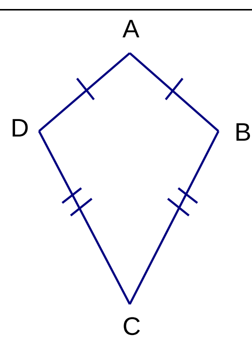
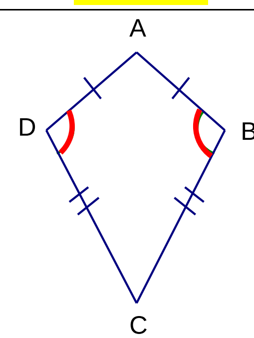
A trapezoid is isosceles IFF its base angles are congruent.	
	
If $\overline{EH} \cong \overline{FG}$,	**Then** $\angle H \cong \angle G$ AND $\angle E \cong \angle F$
If $\angle H \cong \angle G$ OR $\angle E \cong \angle F$,	**Then** $\overline{EH} \cong \overline{FG}$.

A trapezoid is isosceles IFF its diagonals are congruent.	
	
If $\overline{HF} \cong \overline{EG}$,	**Then** trap*EFGH* is isosceles.
If trap*EFGH* is isosceles,	**Then** $\overline{HF} \cong \overline{EG}$.

Midsegment Theorem for Trapezoids

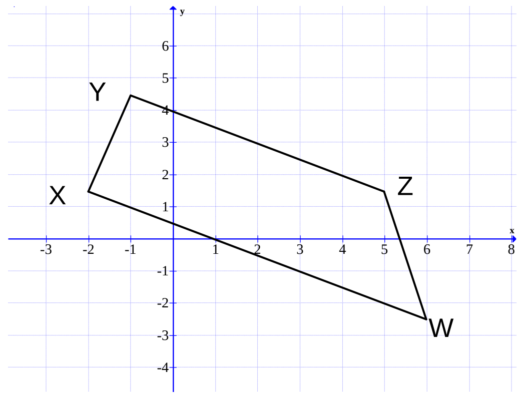
If	**Then**
A midsegment is drawn in a trapezoid,	it is parallel to each base AND its length is one half the sum of the lengths of the bases.
$\overline{MP}$ is the midsegment of a trap$ABCD$	$\overline{MP} \parallel \overline{AB} \parallel \overline{DC}$, AND $MP = \frac{1}{2}(AB + CD)$
	

If	**Then**
a quadrilateral is a kite,	its diagonals are perpendicular.
kite$ABCD$	$\overline{AC} \perp \overline{BD}$
	

If	**Then**
a quadrilateral is a kite,	*exactly* one pair of opposite angles is congruent.
kite$ABCD$	$\angle D \cong \angle B$
	

Show:

Ex1: Show that $XYZW$ is a trapezoid.

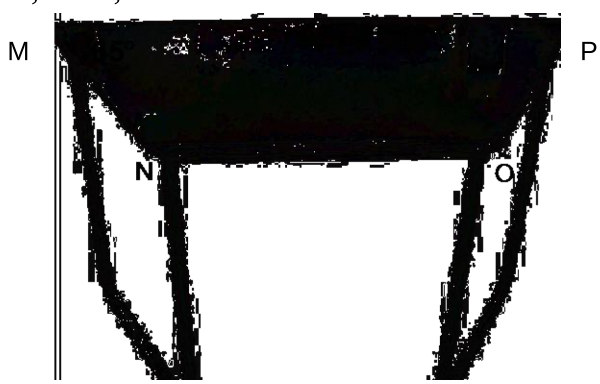


slope $\overline{YZ}$ = slope $\overline{XW} = \frac{1}{2}$

slope $\overline{XY} = 3$ and slope $\overline{ZW} = -4$

Therefore, $\overline{YZ} \parallel \overline{XW}$ and $\overline{XY} \nparallel \overline{ZW}$. Since exactly one pair of sides are parallel, $XYZW$ is a trapezoid.

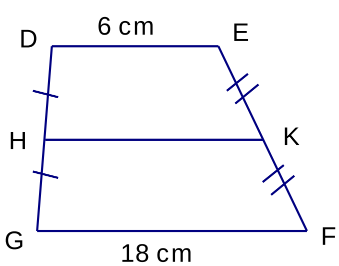
Ex2: The top of the table in the diagram is an isosceles trapezoid. Find $m\angle N, m\angle O,$ and $m\angle P$.



$m\angle P = m\angle M = 65°$

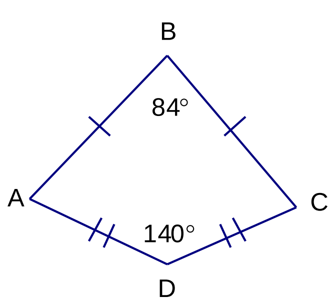
$m\angle N = m\angle O = 360 - 2(65) = 115°$

Ex3: In the diagram, $\overline{HK}$ is the midsegment of the trapezoid $DEFG$. Find HK.



$HK = \frac{6+18}{2} = 12 \text{ cm}$

Ex4: Find $m\angle C$ in the kite shown.



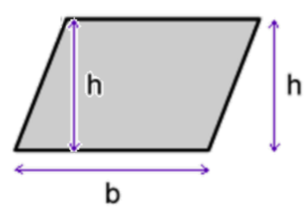
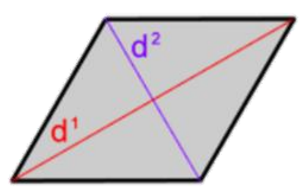
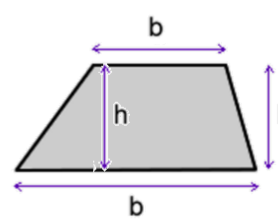
$360 = 140 + 84 + 2m\angle C$

$68° = m\angle C$

Section:	**8 – 7 Area of Special Quadrilaterals**
Essential Question	How can you find areas of special quadrilaterals?

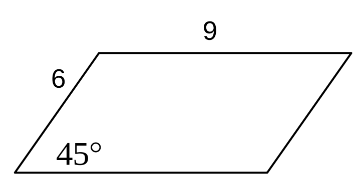
Warm Up:

Formulas:

Area of a Parallelogram	$A = bh$	
Area of a Rhombus	$A = \frac{1}{2}d_1d_2$	
Area of a Trapezoid	$A = \frac{1}{2}h(b_1 + b_2)$	

Show:

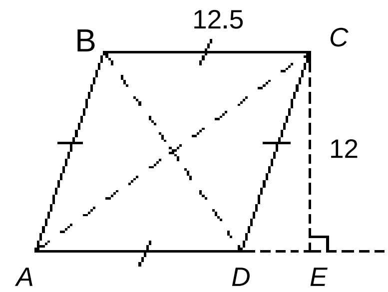
Ex1: Find the area of the parallelogram.



$h = 6\sin 45°$

$A = (6\sin 45°)(9) \approx 38.2$

Ex2: In the rhombus $ABCD$, $AC = 20$ and $BD = 15$. The area can be found in more than one way. Fill in the blanks for each formula, then compute the area.



a) $A =$ _12.5__ $\bullet 12 =$ _150__

Uses parallelogram area formula

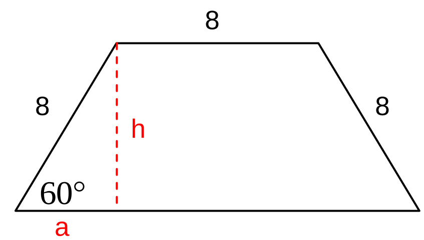
b) $A = \frac{1}{2} \bullet$ __20___ $\bullet$ _15____ $=$ _150___

Uses rhombus area formula

c) $A = 4 \bullet \frac{1}{2} \bullet$ _12.5_ $\bullet$ _6____ $=$ _150__

Uses triangle area formula

Ex3: Find the area of the trapezoid.



$h = 8\sin 60°$

$a = 8\cos 60° = 4$

$b_2 = 8 + 4 + 4$

$A = \frac{1}{2}(8\sin 60°)(8+16)$

$A \approx 83.1$