CHAPTER 6 – SIMILARITY

*In this chapter we address…***Big IDEAS***:*

1) Using ratios and proportions to solve geometry problems
2) Showing that triangles are similar
3) Using indirect measurement and similarity

Section:	**6 – 0 Ratios and Proportions**
Essential Question	**What are the properties of proportions?**

Warm Up:

Key Vocab:

Ratio	A comparison of two numbers. Written as a fraction (never a decimal) or using a colon. Should always be written in simplest form **Examples:** $\frac{2}{3}$ or $2:3$
Proportion	An equation that states two ratios are equal **Examples:** $\frac{2}{3}=\frac{4}{6}$ or $5:7=15:21$
Extended Proportion	An equation relating three or more ratios **Examples:** $\frac{2}{3}=\frac{4}{6}=\frac{6}{9}$

Properties:

Extremes Means Property	
The product of the means is equal to the product of the extremes Think cross multiply	
If $\frac{2}{3}=\frac{4}{6}$	Then $2(6)=3(4)$

Properties of Proportions	
If you interchange the means (or the extremes),	**Then** you get an equivalent proportion
If $\frac{2}{3}=\frac{4}{6}$	**Then** $\frac{2}{4}=\frac{3}{6}$ **OR** $\frac{6}{3}=\frac{4}{2}$
If You take the reciprocals of both the ratios,	**Then** you get an equivalent proportion
If $\frac{2}{3}=\frac{4}{6}$	**Then** $\frac{3}{2}=\frac{6}{4}$

Show:

Ex 1: Express the ratio in simplest form.

a. $\frac{50}{35}=\frac{10}{7}$

b. $26:91=2:7$

Ex 2: Find the measure of each angle.

a. Two complementary angles have measure in ratio 2:3

$$2x+3x=90 \qquad x=18$$

$$2x=36^\circ \qquad 3x=54^\circ$$

b. The measures of the angles of a triangle are in the ratio 2:2:5.

$$2x+2x+5x=180 \qquad x=20$$

$$2x=40^\circ \qquad 5x=100^\circ$$

Ex 2 continued: Find the length of each side.

c. The perimeter of a triangle is 48 cm and the lengths of the sides are in the ratio 3:4:5. Find the length of each side.

$$3x + 4x + 5x = 48$$
$$x = 4$$

$$3x = 12 \text{ cm}$$
$$4x = 16 \text{ cm}$$
$$5x = 20 \text{ cm}$$

Ex 3: Use the properties of proportions to complete each statement.

a. If $\frac{x}{5} = \frac{3}{2}$, then $2x = 15$

b. If $\frac{x}{7} = \frac{y}{8}$, then $\frac{x}{y} = \frac{7}{8}$

c. If $\frac{8}{y} = \frac{3}{x}$, then $\frac{y}{8} = \frac{x}{3}$

d. If $\frac{7}{x} = \frac{3}{4}$, then $3x = 28$

e. If $4 : x = y : 8$, then $8 : x = y : 4$

f. If $4 : x = y : 8$, then $xy = 32$

Ex 4: Find the value of *x*.

a. $\frac{6}{x} = \frac{3}{5}$

$$30 = 3x$$
$$10 = x$$

b. $\frac{x+2}{12} = \frac{1}{2}$

$$2x + 4 = 12$$
$$2x = 8$$
$$x = 4$$

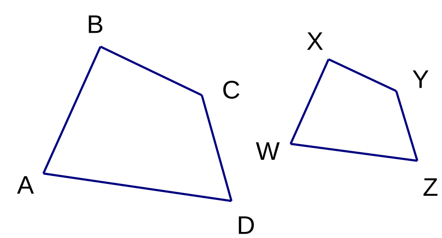
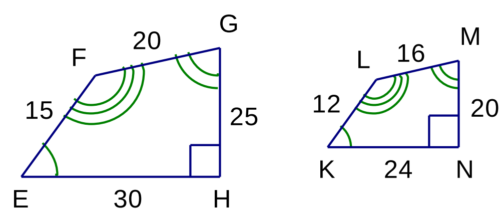
c. $\frac{x+3}{x-5} = \frac{5}{2}$

$$2x + 6 = 5x - 25$$
$$31 = 3x$$
$$\frac{31}{3} = x$$

Section:	**6 – 1 Use Similar Polygons**
Essential Question	**If two figures are similar, how do you find the length of a missing side?**

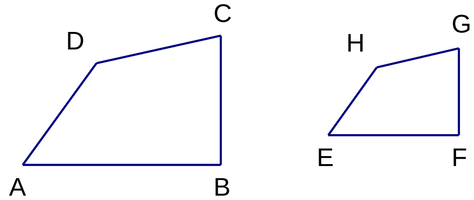
Warm Up:

Key Vocab:

Similar Polygons	Two polygons such that their corresponding angles are **congruent** and the lengths of corresponding sides are **proportional**	 *ABCD* ~ *WXYZ*
Scale Factor	The **ratios** of the lengths of two corresponding sides of two similar polygons.	 The scale factor of *EFGH* to *KLMN* is $\frac{5}{4}$

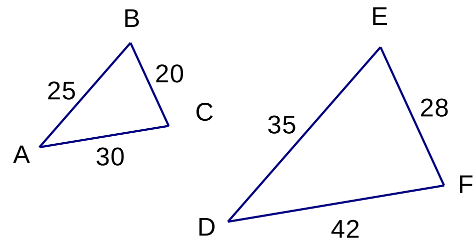
Theorems:

Perimeters of Similar Polygons	
If two polygons are similar,	**then** the ratio of their perimeters is **equal** to the ratios of their corresponding side lengths.
$ABCD \sim EFGH$	$\frac{AB+BC+CD+DA}{EF+FG+GH+HE}=\frac{AB}{EF}=\frac{BC}{FG}=\frac{CD}{GH}=\frac{DA}{HE}$



Show:

Ex 1: In the diagram, $\Delta ABC \sim \Delta DEF$.



a.) List all pairs of congruent angles.

$\angle A \cong \angle D$

$\angle B \cong \angle E$

$\angle C \cong \angle F$

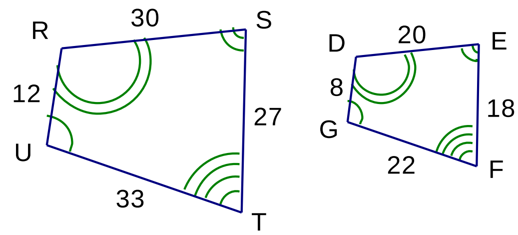
b.) Check that the ratios of corresponding side lengths are equal.

$\frac{AB}{DE}=\frac{5}{7}$ $\quad \frac{BC}{EF}=\frac{5}{7}$ $\quad \frac{AC}{DF}=\frac{5}{7}$

c.) Write the ratios of the corresponding side lengths in a statement of proportionality.

$\frac{AB}{DE}=\frac{BC}{EF}=\frac{AC}{DF}$

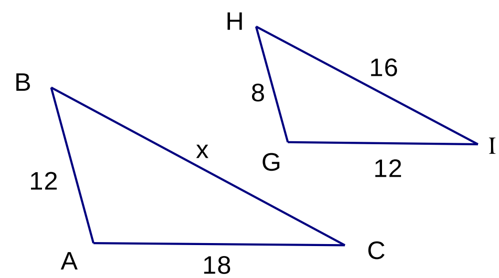
Ex 2: Determine whether the polygons are similar. If they are, write a similarity statement and find the scale factor of *RSTU* to *DEFG*.



All corresponding angles are congruent AND all corresponding sides are proportional.
RSTU~DEFG. The scale factor is $3:2$

Ex 2: In the diagram, $\Delta ABC \sim \Delta GHI$. Find the value of *x*.

$$\frac{12}{8} = \frac{x}{16}$$
$$x = 24$$



Ex 4: You are constructing a rectangular play area. A playground is rectangular with length 25 meters and width 15 meters. The new area will be similar in shape, but only 10 meters in length.

a.) Find the scale factor of the new play area to the playground.

2:5

b.) Find the perimeter of the playground and the play area.

80 m and 32 m

Corresponding Lengths in Similar Polygons	If two polygons are similar, then the ratio of any two corresponding lengths in the polygons is equal to the scale factor of the similar polygons

Ex 3: In the diagram, $\Delta MNP \sim \Delta RST$. Find the length of the altitude $\overline{NL}$.

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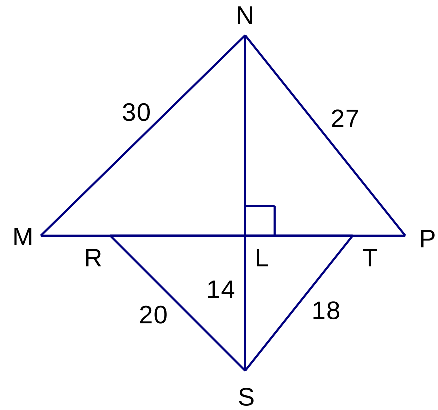
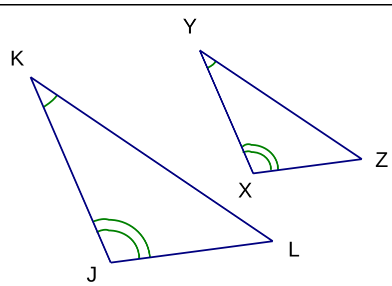
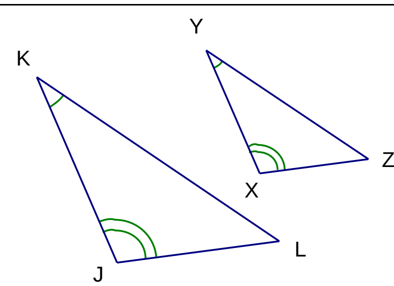


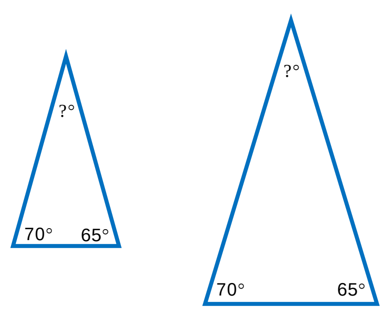
Section:	**6 – 3 Prove Triangles Similar by AA~**
Essential Question	How can you show that two triangles are similar?

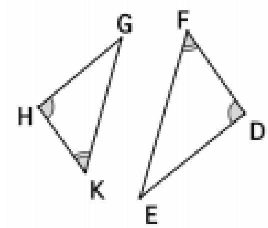
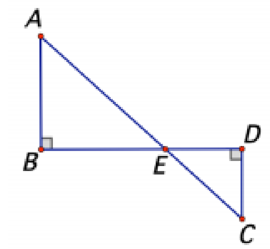
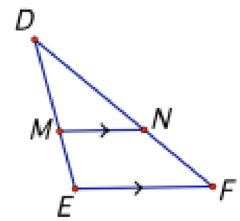
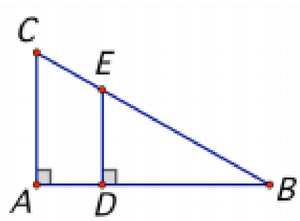
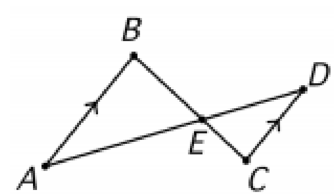
Warm Up:

Postulates:

Angle-Angle (AA~) Similarity Postulate	
If two angles of one triangle are congruent to two angles of another triangle,	**then** the two triangles are similar
$\angle K \cong \angle Y$ and $\angle J \cong \angle X$	$\Delta JKL \sim \Delta XYZ \rightarrow \angle L \cong \angle Z, \frac{KJ}{XY} = \frac{JL}{XZ} = \frac{KL}{YZ}$
	

- If two angles in a triangle are congruent to two angles in another, then the 3rd angles are congruent.
- Therefore, two pair of congruent angles is enough to show that all angles are congruent, so the triangles have the *same shape*, meaning that they are similar.



<table>
<tr><th colspan="2">Things to look for when using the AA~ Postulate:</th></tr>
<tr><td colspan="2">Two pair of angles that are marked congruent
</td></tr>
<tr><td>Vertical angles are always congruent.
</td><td>When lines are parallel, corresponding angles are congruent.
</td></tr>
<tr><td>Reflexive angles are always congruent.
</td><td>When lines are parallel, alternate Interior angles are congruent.
</td></tr>
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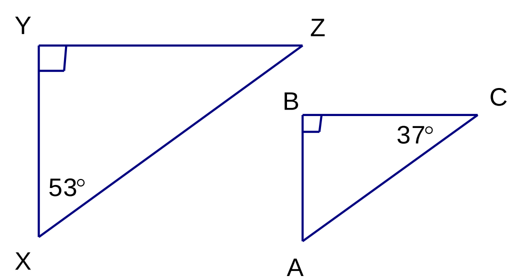
Show:

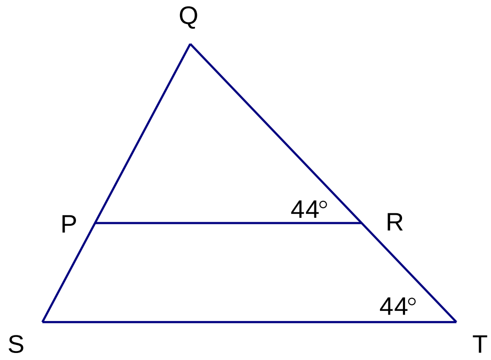
Ex1 : Determine whether the triangles are similar. If they are, write a similarity statement. Explain your reasoning.



$\angle Y \cong \angle B$ because right angles are congruent. By the *Triangle Sum Theorem,* $m\angle Z = 37$, so $m\angle Z = m\angle C$. $\therefore \Delta ABC \sim \Delta XYZ$ by the *AA Similarity Post.*

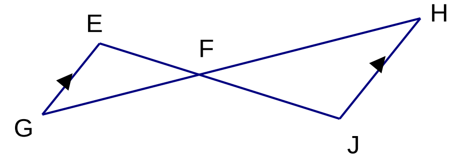
Ex2 : Show that the two triangles are similar.

a.)



$m\angle QRP = 44$ and $m\angle QTS = 44$, so $\angle QRP \cong \angle QTS$. Also, $\angle Q \cong \angle Q$.
$\therefore \Delta QRP \sim \Delta QTS$ by the *AA Similarity Post.*

b.)



$\angle EFG \cong \angle JFH$ because vertical angles are congruent. Also, since $\overline{EG} \parallel \overline{JH}$, $\angle E \cong \angle J$ by the *Alternate Interior Angle Theorem.* $\therefore$ $\Delta EFG \sim \Delta JFH$ by the *AA Similarity*

Ex3 : A school building casts a shadow that is 26 feet long. At the same time a student standing nearby, who is 71 inches tall casts a shadow that is 48 inches long. How tall is the building to the nearest foot?

A. 18 ft

B. 33 ft

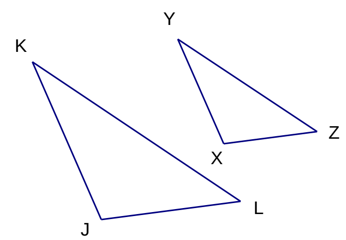
C. 38 ft

D. 131 ft

Section:	**6 – 4 Prove Triangles Similar by SSS~ and SAS~**
Essential Question	How do you prove that two triangles are similar by SSS~ and SAS~?

Warm Up:

Theorems:



Side-Side-Side (SSS~) Similarity Theorem	
If ALL the corresponding side lengths of two triangles are proportional,	**then** the triangles are similar.
$\frac{JK}{XY} = \frac{KL}{YZ} = \frac{LJ}{ZX}$	$\Delta JKL \sim \Delta XYZ \rightarrow$ $\angle J \cong \angle X, \angle K \cong \angle Y, \angle L \cong \angle Z$

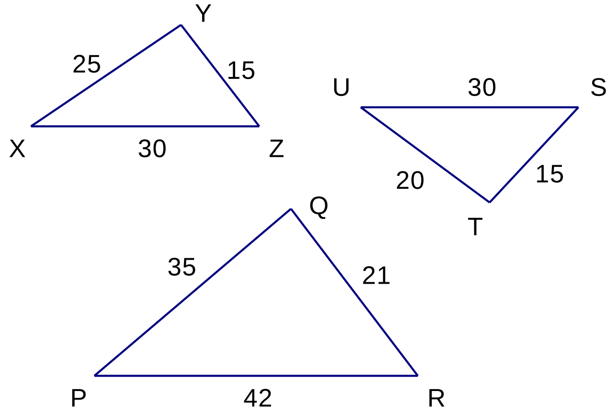
Side-Angle-Side (SAS~) Similarity Theorem	
If an angle of one triangle is congruent to an angle of a second triangle AND the lengths of the included sides are proportional,	**then** the triangles are similar.
$\angle K \cong \angle Y$ AND $\frac{JK}{XY} = \frac{KL}{YZ}$	$\Delta JKL \sim \Delta XYZ \rightarrow$ $\angle J \cong \angle X, \angle L \cong \angle Z, \frac{JK}{XY} = \frac{KL}{YZ} = \frac{JL}{XZ}$

Show:

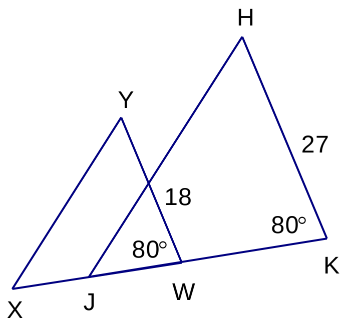
Ex 1: Is either ΔPQR or ΔSTU similar to ΔXYZ ?

$\Delta XYQ \sim \Delta PQR$ by SSS~

ΔXYZ is not similar to ΔSTU .



Ex 2: Is ΔXYW similar to ΔJHK ?.



XW = 16, JK = 24

Yes, by SAS~

$$\frac{18}{16}=\frac{27}{24};\angle JWY \cong \angle K$$

Ex 2: If $\angle Z \cong \angle R$, find the value of x that makes $\Delta XYZ \sim \Delta PQR$.

$$\frac{x+6}{21}=\frac{12}{28}$$
$$28x+168=252$$
$$28x=84$$
$$x=3$$