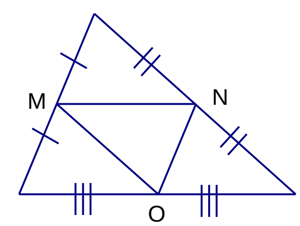
CHAPTER 5 – RELATIONSHIPS WITHIN TRIANGLES

In this chapter we address three **Big IDEAS***:*

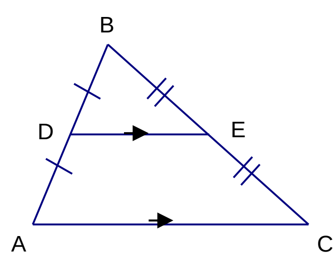
1) Using properties of special segments in triangles
2) Using triangle inequalities to determine what triangles are possible
3) Extending methods for justifying and proving relationships

Section:	**5 – 1 Midsegment Theorem**
Essential Question	**What is a midsegment of a triangle?**

Key Vocab:

Midsegment of a Triangle	A segment that connects the midpoints of two sides of the triangle. **Example:** $\overline{MO}, \overline{MN}, \overline{NO}$ are *midsegments*	

Theorem:

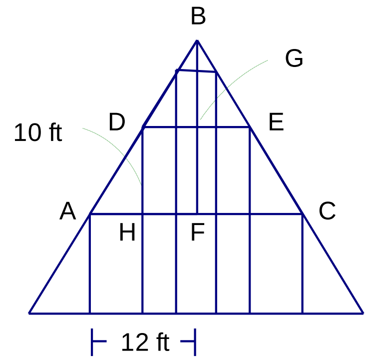
Midsegment Theorem	
The segment connecting the midpoints of two sides of a triangle is a. parallel to the third side b. and is half as long as that side.	 $\overline{DE} \parallel \overline{AC}$ and $DE = \frac{1}{2}AC$

Show:

Ex 1: In the diagram of an A-frame house, $\overline{DG}$ and $\overline{DH}$ are midsegments of ΔABC. Find DG and BF.

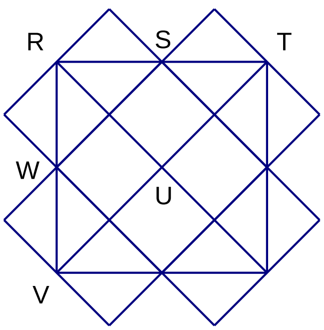
$DG = 6$ ft

$BF = 20$ ft

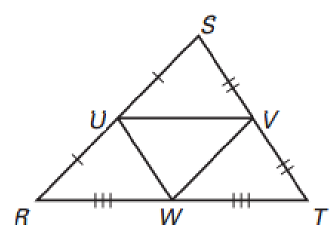


Ex 2: In the diagram, $\overline{RS} \cong \overline{TS}$ and $\overline{RW} \cong \overline{VW}$. Explain why $\overline{VT} \parallel \overline{WS}$

$\overline{RS} \cong \overline{TS}$ and $\overline{RW} \cong \overline{VW}$, so S and W are the midpoints of $\overline{RT}$ and $\overline{RV}$, and $\overline{SW}$ is a midsegment of ΔRTV. Therefore, $\overline{VT} \parallel \overline{WS}$ by the *Midsegment Theorem.*



Ex 3: Use ΔRST to answer each of the following



a. If If $UV = 13$, find RT.

$RT = 2 \bullet 13 = 26$

b. If the perimeter of $\Delta RST = 68$ inches, find the perimeter of ΔUVW.

$\text{Perimeter}_{\Delta UVW} = \frac{1}{2} \bullet 68 = 34 \text{ inches}$

c. If $VW = 2x - 4$ and $RS = 3x - 3$, what is VW?

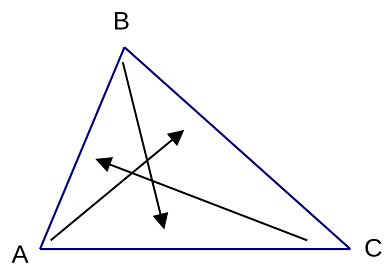
$2(2x-4) = 3x-3$

$4x - 8 = 3x - 3$

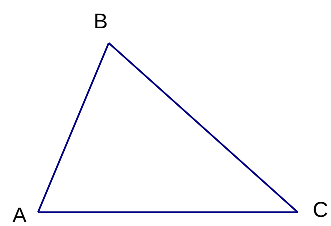
$x = 5$

$VW = 2(5) - 4 = 6$

Section:	**5 – 6 Inequalities in Two Triangles and Indirect Proof**
Essential Question	How do you compare side lengths in triangles?

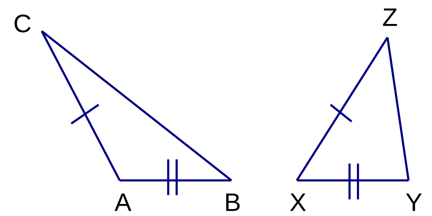
Opposite Side		$\overline{AB}$ is *opposite* $\angle C$ $\overline{BC}$ is *opposite* $\angle A$ $\overline{AC}$ is *opposite* $\angle B$

Theorems:



If one side of a triangle is longer than another side,	**then** the angle opposite the longer side is larger than the angle opposite the shorter side.
$AC > AB$	$m\angle B > m\angle C$
If one angle of a triangle is larger than another angle,	**then** the side opposite the larger angle is longer than the smaller angle.
$m\angle B > m\angle A$	$AC > BC$

Theorems:



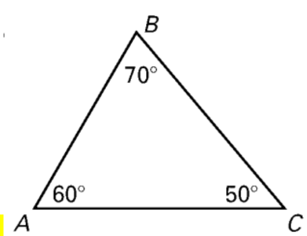
Hinge Theorem (SAS Inequality Theorem)	
If two sides of one triangle are congruent to two sides of another triangle, and the included angle of the first is larger than the included angle of the second,	**then** the third side of the first is longer than the third side of the second.
$\overline{AC} \cong \overline{XZ}, \overline{AB} \cong \overline{XY}$, AND $m\angle A > m\angle X$	$BC > YZ$

Converse of the Hinge Theorem (SSS Inequality Theorem)	
If two sides of one triangle are congruent to two sides of another triangle, and the third side of the first is longer than the third side of the second	**then** the included angle of the first is larger than the included angle of the second.
$\overline{AC} \cong \overline{XZ}, \overline{AB} \cong \overline{XY}$, AND $BC > YZ$	$m\angle A > m\angle X$

Show:

Ex 1: List the sides AND angles in order from smallest to largest.

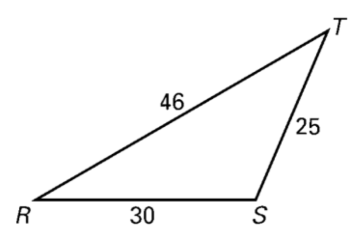
a.



Sides: $\overline{AB}, \overline{BC}, \overline{AC}$

Angles: $\angle C, \angle A, \angle B$

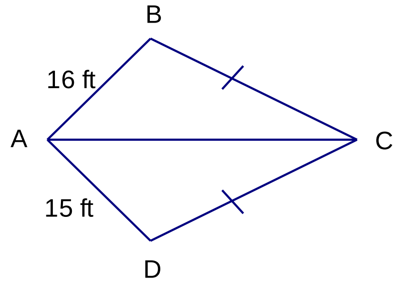
b.



Sides: $\overline{ST}, \overline{RS}, \overline{RT}$

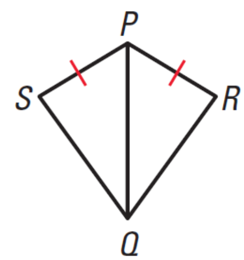
Angles: $\angle R, \angle T, \angle S$

Ex 2: Given that $\overline{BC} \cong \overline{DC}$, how does $\angle ACB$ compare to $\angle ACD$?



$\angle ACB > \angle ACD$ by the SSS Inequality (Hinge Converse) Theorem

Ex 3:



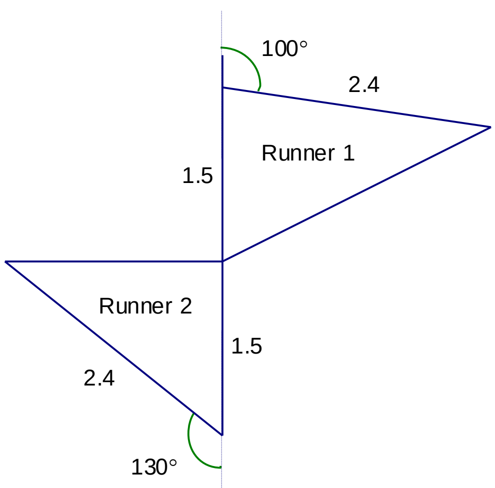
a. If $PR = PS$ and $m\angle QPR > m\angle QPS$, which is longer: $\overline{SQ}$ or $\overline{RQ}$?

$\overline{RQ}$ by the SAS Inequality (Hinge) Theorem

b. If $PR = PS$ and $RQ < SQ$, which is larger: $\angle RPQ$ or $\angle SPQ$?

$\angle SPQ$ by the SSS Inequality (Hinge Converse) Theorem

Ex 4: Two runners start together and run in opposite directions. Each one goes 1.5 miles, changes direction, and goes 2.4 miles. The first runner starts due north and runs. 100° towards the east. The other runner starts due south and turns 130° towards the west. Both runners return to the starting point. Which runner ran farther? *Explain.*



Each triangle has side lengths 1.5 mi and 2.4 mi, and the angles between those sides are 80° and 50°. The *Hinge Theorem*, the third side of the triangle for Runner 1 is longer, so Runner 1 ran further.

Key Vocab:

Indirect Proof	A proof in which you prove that a statement is true by first assuming that its opposite is true. If this assumption leads to an impossibility or contradiction, then you have proved that the original statement is true.

Key Concept:

How to Write an Indirect Proof:

1. Identify the statement you want to prove. Assume temporarily that this statement is false by assuming that its opposite is true.
2. Reason logically until you reach a contradiction
3. Point out that the desired conclusion must be true because the contradiction proves the temporary assumption false.

Ex 5: Write an indirect proof.

Given: $2r+3 \neq 17$

Prove: $r \neq 7$

Temporarily assume that $r = 7$. If $r = 7$, then $2r + 3 = 2(7) + 3 = 17$. However, this conclusion contradicts the given information that $2r + 3 \neq 17$. Therefore, the assumption was incorrect and it follows that $r \neq 7$.

Ex 6: Write an indirect proof.

Given: $m\angle X \neq m\angle Y$
Prove: $\angle X$ and $\angle Y$ are not both right angles

Let's assume temporarily that $\angle X$ and $\angle Y$ are actually right angles. Right angles are defined to contain $90°$, so $m\angle X = 90°$ and $m\angle Y = 90°$. Then, by the Transitive Property, $m\angle X = m\angle Y$. But this equality contradicts the given statement that $m\angle X \neq m\angle Y$. So our initial assumption must have been false, meaning $\angle X$ and $\angle Y$ cannot both be right angles.

Closure:

- Describe the difference between the Hinge Theorem and its Converse.

The Hinge Theorem is also called the SAS Inequality Theorem and it makes a conclusion about the third side of a triangle.

The Converse of the Hinge Theorem is also called the SSS Inequality Theorem and it makes a conclusion about the included angle of a triangle.