CHAPTER 7 – RIGHT TRIANGLES & TRIGONOMETRY

Big IDEAS*:*

1) ***Using Pythagorean Theorem and its converse.***
2) ***Using special relationships in right triangles.***
3) ***Using trigonometric ratios to solve right triangles.***

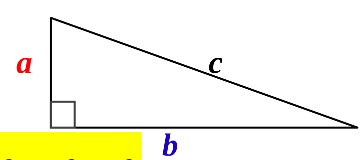
Section:	**7 – 1 Applying the Pythagorean Theorem**
Essential Question	If you know the lengths of two sides of a right triangle, how do you find the length of the third side?

Warm Up:

Key Vocab:

Pythagorean Triple	A set of three positive integers a, b, and c that satisfy the equation $c^2 = a^2 + b^2$	The most common are: 3, 4, 5 5, 12, 13 8, 15, 17 7, 24, 25

Theorems:

Pythagorean Theorem	
In a right triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the legs.	 $c^2 = a^2 + b^2$

Show:

Ex 1: Find the length of the hypotenuse of a right triangle with legs measuring 5 and 12.

$c^2 = a^2 + b^2$

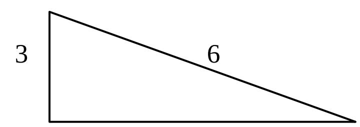
$c^2 = 5^2 + 12^2$

$c^2 = 25 + 144$ The hypotenuse is 13 units.

$\sqrt{c^2} = \sqrt{169}$

$\boxed{c = 13}$

Ex 2: Randy made a ramp for his dog to get into his truck. The ramp is 6 feet long and the bed of the truck is 3 feet above the ground. Approximately how far from the back of the truck does the ramp touch the ground?



$c^2 = a^2 + b^2$

$6^2 = 3^2 + b^2$

$36 = 9 + b^2$

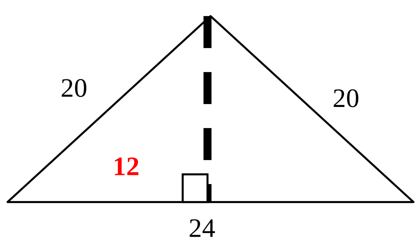
$36 - 9 = 9 - 9 + b^2$

$\sqrt{27} = \sqrt{b^2}$

$b = \sqrt{9 \cdot 3}$

$\boxed{b = 3\sqrt{3} \approx 5.2 \text{ ft}}$

Ex 3: Find the area of an isosceles triangle with side lengths 20 inches, 20 inches and 24 inches.



$c^2 = a^2 + b^2$

$20^2 = 12^2 + b^2$

$400 = 144 + b^2$

$400 - 144 = 144 - 144 + b^2$

$\sqrt{256} = \sqrt{b^2}$

$b = \sqrt{256}$

$\boxed{b = 16 \text{ in}}$

$A = \frac{1}{2}bh$

$A = \frac{1}{2}(16)(24)$

$A = \boxed{192 \text{ in}^2}$

Ex 4: The given lengths are two sides of a right triangle. All three side lengths of the triangle are integers and together form a Pythagorean triple. Find the length of the third side and tell whether it is a leg or the hypotenuse.

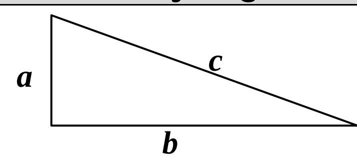
a. 12, 13	**b.** 21, 72	**c.** 9, 15
5 – leg	75 – hypotenuse	12 – leg

Section:	**7 – 2 Use the Converse of the Pythagorean Theorem**
Essential Question	How can you use the sides of a triangle to determine if it is right triangle?

Warm Up:

Theorems:

Converse of the Pythagorean Theorem



If	**then**
the square of the length of the longest side of a triangle is equal to the sum of the squares of the lengths of the two shorter sides,	the triangle is a right triangle.
$c^2 = a^2 + b^2$	
If	**then**
the square of the length of the longest side of a triangle is less than the sum of the squares of the lengths of the two shorter sides,	the triangle is an acute triangle.
$c^2 < a^2 + b^2$	
If	**then**
the square of the length of the longest side of a triangle is greater than to the sum of the squares of the length of the two shorter sides,	the triangle is an obtuse triangle.
$c^2 > a^2 + b^2$	

Show:

Ex 1: Tell if the given triangle is right, acute, or obtuse – sides are 11, 20, 23.

$c^2 = a^2 + b^2$

$23^2 \stackrel{?}{=} 11^2 + 20^2$

$529 \stackrel{?}{=} 121 + 400$

$529 \stackrel{?}{=} 521$

$\boxed{529 \neq 521}$

$c^2 \stackrel{?}{=} a^2 + b^2$

$\boxed{529 > 521}$

Since $c^2 > a^2 + b^2$, the triangle is obtuse.

Ex 2: Tell if the given triangle is right, acute, or obtuse – sides are $10, 8, 2\sqrt{41}$.

$c^2 = a^2 + b^2$

$\left(2\sqrt{41}\right)^2 \stackrel{?}{=} 8^2 + 10^2$

$164 \stackrel{?}{=} 64 + 100$

$\boxed{164 \stackrel{?}{=} 164}$

Since $c^2 = a^2 + b^2$, the triangle is right.

Ex 3: The sides of a triangle have lengths x, $x - 8$, 40. If the length of the longest side is 40, what values of x will make the triangle acute?

$c^2 < a^2 + b^2$

$40^2 < x^2 + (x-8)^2$

$1600 < x^2 + x^2 - 16x + 64$

$0 < 2x^2 - 16x - 1536$

$0 < x^2 - 8x - 768$

$0 < (x+24)(x-32)$

~~$x = -24$~~ $x = 32$

$32 < x < 40$

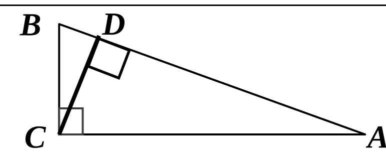
Section:	**7 – 3 Use Similar Right Triangles**
Essential Question	How can you find the length of the altitude to the hypotenuse of a right triangle?

Warm Up:

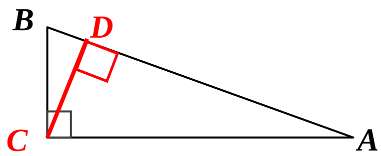
Key Vocab:

Geometric Mean	For two positive numbers ***a*** & ***b***, the positive number ***x*** that satisfies $\frac{a}{x}=\frac{x}{b}$ So, $x^2=ab$ and $x=\sqrt{ab}$. **Example:** $\frac{4}{x}=\frac{x}{16}$ $x=\sqrt{4\cdot 16}$ $\boxed{x=8}$

Theorems:

If the altitude is drawn to the hypotenuse of a right triangle,	**Then** the two triangles formed are similar to the original triangle and to each other.	 $\Delta CBD\sim\Delta ABC\sim\Delta ACD$

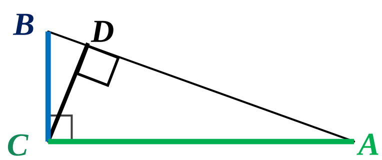
Geometric Mean (Altitude) Theorem



$$\frac{BD}{CD} = \frac{CD}{AD} \qquad \frac{\text{Hyp. Seg.}}{\text{Alt.}} = \frac{\text{Alt.}}{\text{Hyp. Seg.}}$$

If	Then
the altitude is drawn to the hypotenuse of a right triangle,	The length of the altitude is the geometric mean of the lengths of the two segments of the hypotenuse.

Geometric Mean (Leg) Theorem



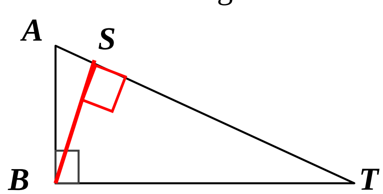
$$\frac{AB}{BC} = \frac{BC}{DB} \qquad \frac{\text{Hyp.}}{\text{Leg}} = \frac{\text{Leg}}{\text{Adj. Hyp. Seg.}}$$

$$\frac{AB}{AC} = \frac{AC}{AD} \qquad \frac{\text{Hyp.}}{\text{Leg}} = \frac{\text{Leg}}{\text{Adj. Hyp. Seg.}}$$

If	Then
the altitude is drawn to the hypotenuse of a right triangle,	The length of each leg of the right triangle is the geometric mean of the lengths of the hypotenuse AND the segment of the hypotenuse that is adjacent to the leg.

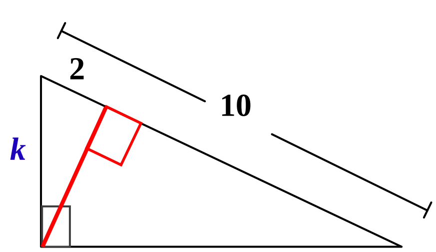
Show:

Ex 1: Identify the similar triangles in the diagram.



$\Delta ABT \sim \Delta ASB \sim \Delta BST$

Ex 2: Find the value of k.



Geometric Mean Leg Theorem

$$\frac{10}{k} = \frac{k}{2}$$

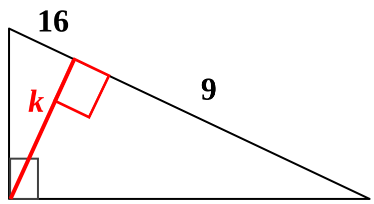
$$k^2 = 20$$

$$\sqrt{k^2} = \sqrt{20}$$

$$k = \sqrt{4}\sqrt{5}$$

$$\boxed{k = 2\sqrt{5}}$$

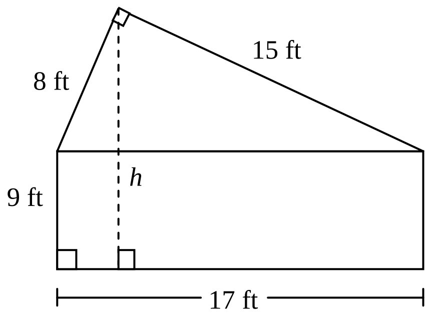
Ex 3: Find the value of *k*.



Geometric Mean Altitude Theorem

$$\frac{16}{k} = \frac{k}{9}$$
$$144 = k^2$$
$$12 = k$$

Ex 4: The figure shows the side view of a tool shed. What is the maximum height of the shed?



Find height of roof triangle:

$$\frac{y}{15} = \frac{8}{17}$$
$$17y = 15(8)$$
$$y = \frac{120}{17}$$
$$y = 7.059$$

$$\text{Total height}(h) = \text{roof height} + 9$$
$$h = 7.059 + 9$$
$$\boxed{h = 16.059 \text{ ft}}$$

Closure:

- Describe the transformation(s) that move the two smaller triangles onto each other.

  The smaller triangles can be rotated onto each other

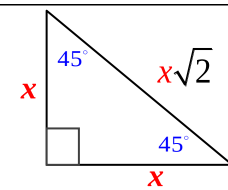
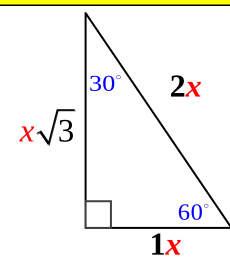
- Describe the transformation(s) that move either of the two smaller triangles onto the whole triangle.

  The smaller triangles need to be reflected and then rotated onto the whole triangle.

Section:	**7 – 4 Special Right Triangles**
Essential Question	How do you find the lengths of the sides of a $30^\circ - 60^\circ - 90^\circ$ triangle and a $45^\circ - 45^\circ - 90^\circ$ triangle?

Warm Up:

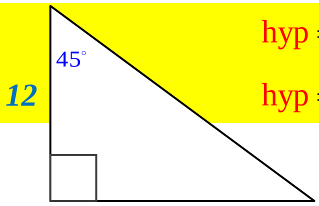
Theorems:

$45^\circ - 45^\circ - 90^\circ$ Triangle	The hypotenuse is $\sqrt{2}$ times as long as each leg. **Easy as …** $1x$, $1x$, $x\sqrt{2}$!	 $\text{hypotenuse} = \text{leg}\sqrt{2}$
$30^\circ - 60^\circ - 90^\circ$ Triangle	The hypotenuse is 2 times as long as the short leg. The longer leg is $\sqrt{3}$ times as long as the shorter leg. **Easy as …** $1x$, $2x$, $x\sqrt{3}$!	

Show:

Ex 1: Find the length of the hypotenuse.

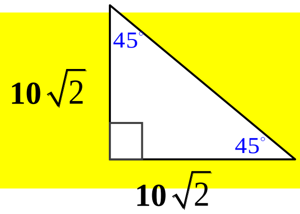
a.



$\text{hyp} = x\sqrt{2}$

$\text{hyp} = \boxed{12\sqrt{2}}$

b.

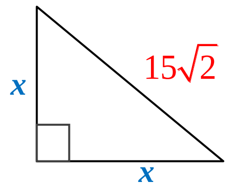


$\text{hyp} = x\sqrt{2}$

$\text{hyp} = \left(10\sqrt{2}\right)\sqrt{2}$

$\text{hyp} = \boxed{20}$

Ex 2: Find the length of the legs in the triangle.

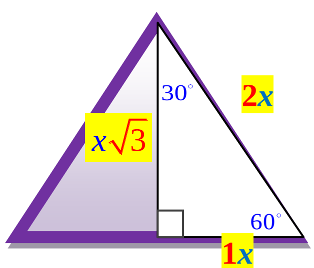


$15\sqrt{2} = x\sqrt{2}$

$\frac{15\sqrt{2}}{\sqrt{2}} = \frac{x\sqrt{2}}{\sqrt{2}}$

$x = \boxed{15}$

Ex 3: Alex has a team logo patch in the shape of an equilateral triangle. If the sides are 2.5 inches long, what is the approximate height of his patch?



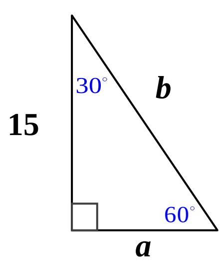
$2x = 2.5$

$x = 1.25$

$x\sqrt{3} = 1.25\sqrt{3}$

$x = \boxed{2.165 \text{ in}}$

Ex 4: Find the values of *a* and *b*. Write your answers in simplest radical form.



$x\sqrt{3} = 15$

$\frac{x\sqrt{3}}{\sqrt{3}} = \frac{15}{\sqrt{3}}$

$x = \frac{15}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$

$x = \frac{15}{3}\sqrt{3}$

$x = \boxed{5\sqrt{3}}$

$a = x$

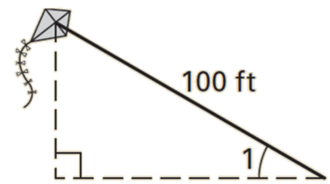
$\therefore a = \boxed{5\sqrt{3}}$

$b = 2x$

$\therefore b = 2\left(5\sqrt{3}\right)$

$b = \boxed{10\sqrt{3}}$

Ex 6: A kite is attached to a 100 foot string as shown. How far above the ground is the kite when the string forms the given angle?



a. $m\angle 1 = 45^\circ$

$$hyp = x\sqrt{2}$$
$$\therefore \frac{x\sqrt{2}}{\sqrt{2}} = \frac{100}{\sqrt{2}}$$
$$x = \frac{100}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$$
$$x = \frac{100\sqrt{2}}{2}$$
$$x = \boxed{50\sqrt{2} \approx 70.71 \text{ ft}}$$

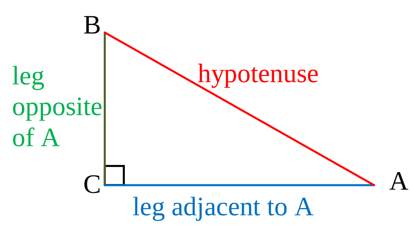
b. $m\angle 1 = 30^\circ$

$$hyp = 2x$$
$$\therefore \frac{2x}{2} = \frac{100}{2}$$
$$x = \boxed{50 \text{ ft}}$$

Section:	**7 – 5 Apply the Tangent Ratio**
Essential Question	How can you find a leg of a right triangle when you know the other leg and one acute angle?

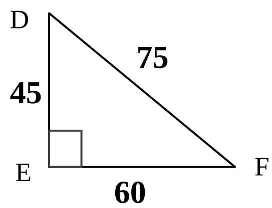
Warm Up:

Key Vocab:

Trigonometric Ratio	The ratio of the lengths of two sides in a right triangle. Three common trigonometric ratios are sine, cosine, and tangent.	
Tangent Ratio	Let ΔABC, be a right triangle with acute angle $\angle A$, then $\tan A = \dfrac{\text{length of leg opposite } \angle A}{\text{length of leg adjacent } \angle A}$	

Show:

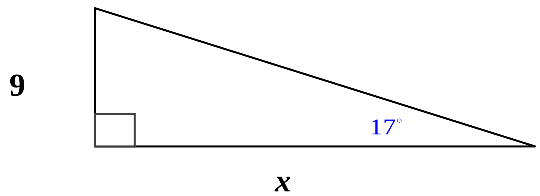
Ex 1: Find the $\tan D$ and the $\tan F$. Write each answer as a fraction and as a decimal rounded to four places.



$$\tan D = \frac{\text{opposite}}{\text{adjacent}} = \frac{60}{45} = \boxed{\frac{4}{3} \approx 1.3333}$$

$$\tan F = \frac{\text{opposite}}{\text{adjacent}} = \frac{45}{60} = \boxed{\frac{3}{4} \approx 0.7500}$$

Ex 2: Find the value of x.



$$\tan 17^\circ = \frac{\text{opposite}}{\text{adjacent}}$$

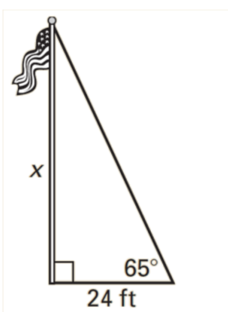
$$\tan 17^\circ = \frac{9}{x}.$$

$$x \tan 17^\circ = 9$$

$$x = \frac{9}{\tan 17^\circ}$$

$$\boxed{x \approx 29.438}$$

Ex 3: Find the height of the flagpole to the nearest foot.



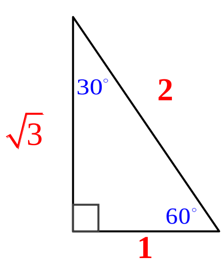
$$\tan 65^\circ = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan 65^\circ = \frac{x}{24}$$

$$x = 24 \tan 65^\circ$$

$$x = \boxed{51 \text{ ft}}$$

Ex 4: What is the exact value of the tangent ratio of a 30° angle?



$$\tan 30^\circ = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$$

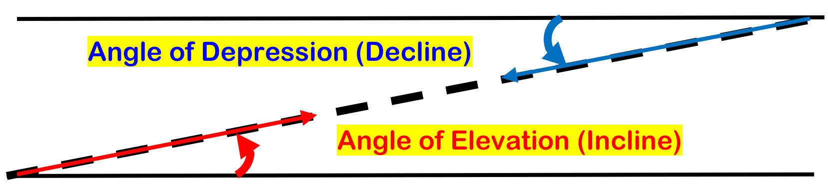
$$\tan 30^\circ = \boxed{\frac{\sqrt{3}}{3}}$$

Section:	**7 – 6 Apply the Sine and Cosine Ratios**
Essential Question	How can you find the lengths of the sides of a right triangle when you are given the length of the hypotenuse and one acute angle?

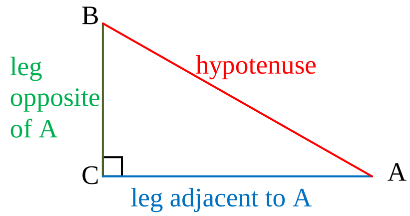
Warm Up:

Key Vocab:

Angle of Elevation (Incline)	The angle of sight when looking up at an object
Angle of Depression (Decline)	The angle of sight when looking down at an object



Key Concepts:



Sine Ratio	Let ΔABC, be a right triangle with acute angle $\angle A$, then $\sin A = \dfrac{\text{length of leg opposite } \angle A}{\text{hypotenuse}} = \dfrac{\text{opposite}}{\text{hypotenuse}}$
Cosine Ratio	Let ΔABC, be a right triangle with acute angle $\angle A$, then $\cos A = \dfrac{\text{length of leg adjacent } \angle A}{\text{hypotenuse}} = \dfrac{\text{adjacent}}{\text{hypotenuse}}$

The great Chief of the Trigonometry Tribe SOH CAH TOA is a good memory device …

$\sin A = \dfrac{\text{opposite}}{\text{hypotenuse}}$

$\cos A = \dfrac{\text{adjacent}}{\text{hypotenuse}}$

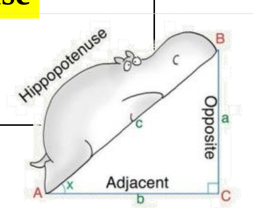
$\tan A = \dfrac{\text{opposite}}{\text{adjacent}}$

Another memory device … hippos

Orange Hippos $\rightarrow \sin A = \dfrac{\textbf{opposite}}{\textbf{hypotenuse}}$

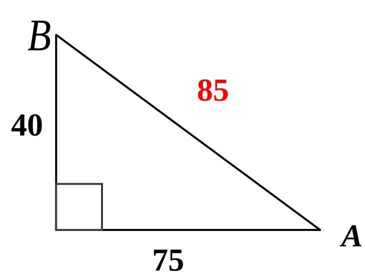
Always Have $\rightarrow \cos A = \dfrac{\textbf{adjacent}}{\textbf{hypotenuse}}$

Orange Ankles $\rightarrow \tan A = \dfrac{\textbf{opposite}}{\textbf{adjacent}}$



Show:

Ex 1: Find the $\sin A$ and $\sin B$. Write each answer as a fraction and decimal rounded to four places.



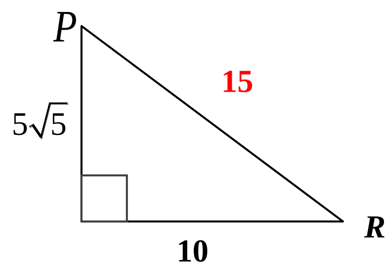
$40^2 + 75^2 = c^2$

$85 = c$

$\sin A = \dfrac{\text{opposite}}{\text{hypotenuse}} = \dfrac{40}{85} = \boxed{\dfrac{8}{17} \approx .4706}$

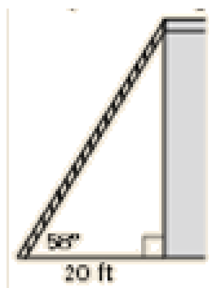
$\sin B = \dfrac{\text{opposite}}{\text{hypotenuse}} = \dfrac{75}{85} = \boxed{\dfrac{15}{17} \approx .8824}$

Ex 2: Find the $\cos P$ and $\cos R$. Write each answer as a fraction and decimal rounded to four places.



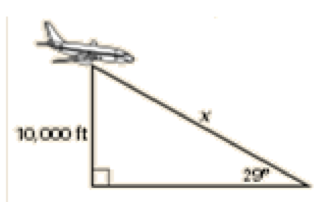
$$10^2 + \left(5\sqrt{5}\right)^2 = c^2$$
$$15 = c$$
$$\cos P = \frac{\text{adj}}{\text{hyp}} = \frac{5\sqrt{5}}{15} = \boxed{\frac{\sqrt{5}}{3} \approx .7454}$$
$$\cos R = \frac{\text{adj}}{\text{hyp}} = \frac{10}{15} = \boxed{\frac{2}{3} \approx .6667}$$

Ex 3: A rope, staked 20 feet from the base of a building, goes to the roof and forms an angle of elevation of 58°. To the nearest tenth of a foot, how long is the rope?



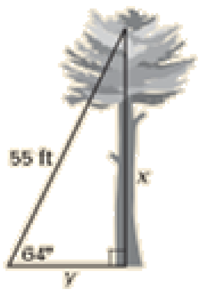
$$\cos 58^\circ = \frac{\text{adj}}{\text{hyp}}$$
$$x\cos 58^\circ = \frac{20}{x}\cdot x$$
$$\frac{x\cos 58^\circ}{\cos 58^\circ} = \frac{20}{\cos 58^\circ}$$
$$x = \frac{20}{\cos 58^\circ} \quad \boxed{\approx 37.7\text{ ft}}$$

Ex 4: A pilot is looking at an airport form her plane. The angle of depression is 29°. If the plane is at an altitude of 10,000 ft, approximately how far is the air distance to the runway?



$$\sin 29^\circ = \frac{\text{opp}}{\text{hyp}}$$
$$x\sin 29^\circ = \frac{10000}{x}\cdot x$$
$$\frac{x\sin 29^\circ}{\sin 29^\circ} = \frac{10000}{\sin 29^\circ}$$
$$x = \frac{10000}{\sin 29^\circ} \quad \boxed{\approx 20626.7\text{ ft}}$$

Ex 5: A dog is looking at a squirrel at the top of a tree. The distance between the two animals is 55 feet and the angle of elevation is 64°. How high is the squirrel and how far is the dog from the base if the tree?



$$\sin 64^\circ = \frac{\text{opp}}{\text{hyp}}$$

$$55\sin 64^\circ = \frac{x}{55}\cdot 55$$

$$x = 55\sin 64^\circ$$

$$\boxed{x \approx 49.4 \text{ ft high}}$$

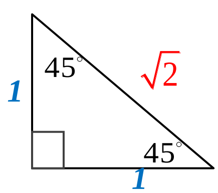
$$\cos 64^\circ = \frac{\text{adj}}{\text{hyp}}$$

$$55\cos 64^\circ = \frac{x}{55}\cdot 55$$

$$x = 55\cos 64^\circ$$

$$\boxed{x \approx 24.1 \text{ ft from the tree}}$$

Ex 6: Use a special right triangle to find the sine and cosine of a 45° angle.



$$\sin 45^\circ = \frac{\text{opp}}{\text{hyp}}$$

$$\sin 45^\circ = \frac{1}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}}$$

$$\sin 45^\circ = \boxed{\frac{\sqrt{2}}{2}}$$

$$\cos 45^\circ = \frac{\text{opp}}{\text{hyp}}$$

$$\cos 45^\circ = \frac{1}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}}$$

$$\cos 45^\circ = \boxed{\frac{\sqrt{2}}{2}}$$

Section:	# 7 – 7 Solve Right Triangles
Essential Question	In a right triangle, how can you find all the sides and angles of the triangle?

Warm Up:

Key Vocab:

Solve a Right Triangle	Find the lengths of all sides and the measures of all angles
Inverse Function	"Opposite Functions" Functions that cancel each other out **Example:** x^2 and $\sqrt{x}$

Key Concepts:

Inverse Sine

This inverse function is used to find the measure of an acute angle in a right triangle when the opposite side and hypotenuse of the triangle are either given or can be determined.

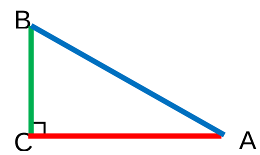
"The angle whose sine value is…"

Notation: $\sin^{-1}$

How it works: if $\sin A = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{BC}{AB}$

$$\cancel{\sin^{-1}}\left(\cancel{\sin} A\right) = \sin^{-1}\left(\frac{BC}{AB}\right)$$

$$m\angle A = \sin^{-1}\left(\frac{BC}{AB}\right)$$



Inverse Cosine

This inverse function is used to find the measure of an acute angle in a right triangle when the adjacent side and hypotenuse of the triangle are either given or can be determined.

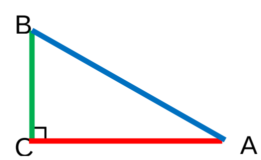
"The angle whose cosine value is…"

Notation: $\cos^{-1}$

How it works: if $\cos A = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{AC}{AB}$

$$\cancel{\cos^{-1}}\left(\cancel{\cos} A\right) = \cos^{-1}\left(\frac{AC}{AB}\right)$$

$$m\angle A = \cos^{-1}\left(\frac{AC}{AB}\right)$$



Inverse Tangent

This inverse function is used to find the measure of an acute angle in a right triangle when the opposite and adjacent sides of the triangle are either given or can be determined.

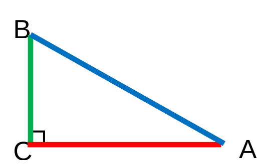
"The angle whose tangent value is…"

Notation: $\tan^{-1}$

How it works: if $\tan A = \frac{\text{opposite}}{\text{adjacent}} = \frac{BC}{AC}$

$$\cancel{\tan^{-1}}\left(\cancel{\tan} A\right) = \tan^{-1}\left(\frac{BC}{AC}\right)$$

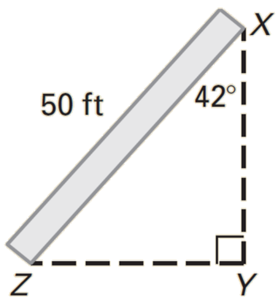
$$m\angle A = \tan^{-1}\left(\frac{BC}{AC}\right)$$



Caution! The notations $\tan^{-1}, \sin^{-1}$, and $\cos^{-1}$ are **different** from a negative exponent meaning to take a reciprocal. **Example:** $x^{-1} = \frac{1}{x}$ **Non-example:** $\tan^{-1} \neq \frac{1}{\tan}$

Show:

Ex 1: Solve the triangle formed by the water slide shown in the figure. Round decimal answers to the nearest tenth. Note: there are multiple, correct solution paths.



$\measuredangle Z = 108^\circ - 90^\circ - 42^\circ$

$\boxed{\measuredangle Z = 48^\circ}$

$\cos Z = \dfrac{\overline{ZY}}{\text{hyp}}$

$50 \cdot \cos 48^\circ = \dfrac{\overline{ZY}}{\cancel{50}} \cdot \cancel{50}$

$\overline{ZY} = 50 \cdot \cos 48^\circ \boxed{\approx 33.5 \text{ ft}}$

$\sin Z = \dfrac{\overline{XY}}{\text{hyp}}$

$50 \cdot \sin 48^\circ = \dfrac{\overline{XY}}{\cancel{50}} \cdot \cancel{50}$

$\overline{XY} = 50 \cdot \sin 48^\circ \boxed{\approx 33.2 \text{ ft}}$

Ex 2: Let $\angle T, \angle K, \angle A$ be an acute angles in right triangles. Use a calculator to approximate the measure of each to the nearest tenth of a degree. *Show your work*.

a. $\cos T = 0.37$

$\cancel{\cos^{-1}}\left(\cancel{\cos} T\right) = \cos^{-1}(0.37)$

$T = \cos^{-1}(0.37)$

$\boxed{T = 68.3^\circ}$

b. $\sin K = 0.24$

$\cancel{\sin^{-1}}\left(\cancel{\sin} K\right) = \sin^{-1}(0.24)$

$K = \sin^{-1}(0.24)$

$\boxed{K = 13.9^\circ}$

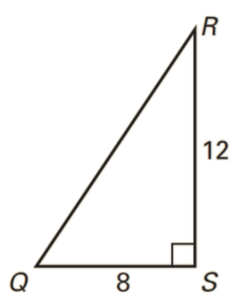
c. $\tan A = 15.85$

$\cancel{\tan^{-1}}\left(\cancel{\tan} A\right) = \tan^{-1}(15.85)$

$A = \tan^{-1}(15.85)$

$\boxed{A = 86.3^\circ}$

Ex 3: Use the calculator to approximate the measure of $\measuredangle Q$ to the nearest tenth of a degree.

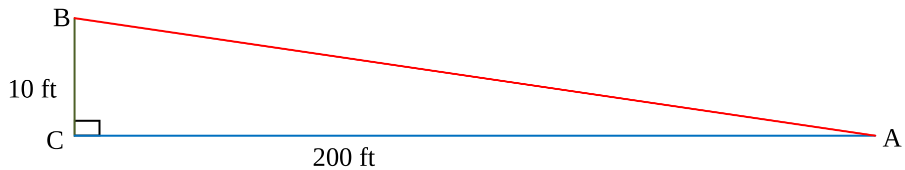


$\cancel{\tan^{-1}}\left(\cancel{\tan} Q\right) = \tan^{-1} \dfrac{\text{opposite}}{\text{adjacent}}$

$Q = \tan^{-1}\left(\dfrac{12}{8}\right)$

$\boxed{Q = 56.3^\circ}$

Ex 4: A road rises 10 feet in a horizontal distance of 200 feet. What is the angle of inclination?



$$\tan^{-1}(\tan A) = \tan^{-1}\left(\frac{10}{200}\right)$$

$$A = \tan^{-1}\left(\frac{1}{20}\right)$$

$$\boxed{A = 2.9^\circ}$$

Closure:

- When do you use Sine, Cosine, and Tangent?
  Sine, cosine, and tangent are used to find a missing side length in a right triangle

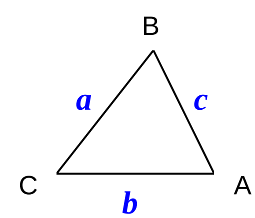
- When do you use Inverse Sine, Inverse Cosine, and Inverse Tangent?
  Inverse Sine, Inverse Cosine, and Inverse Tangent are used to find a missing angle measure in a right triangle.

Section:	**7 - 8 Law of Sines and Law of Cosines**
Essential Question	How do you find sides and angles of oblique triangles?

Warm Up:

Key Vocab:

Oblique Triangles	Triangles that have no right angles – they are either acute or obtuse.
Law of Sines	$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ **OR** $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$
Law of Cosines	$a^2 = b^2 + c^2 - 2bc\cos A$ **OR** $b^2 = a^2 + c^2 - 2ac\cos B$ **OR** $c^2 = a^2 + b^2 - 2ab\cos C$



Oblique ΔABC, where

$\angle A$ and side a are always opposite each other,

$\angle B$ and side b are always opposite each other, and

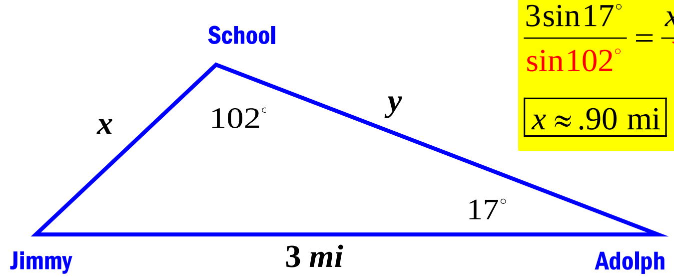
$\angle C$ and side c are always opposite each other

Key Concept:

Solving Oblique Triangles	
Given Information	**Solution Method**
Two angles and any side: AAS or ASA	Law of Sines
<u>* The Ambiguous Case * :</u> **Two sides and a non-included angle:** *SSA* ➢ may have one solution, two solutions, or no solutions.	Law of Sines
Three sides: SSS	Law of Cosines
Two sides and an included angle: SAS	Law of Cosines

Show:

Ex 1: Use the information in the diagram to find the distance each person lives from the school – round answers to the nearest hundredth. Show your work. How much closer to school does Jimmy live compared to Adolph?



$$\frac{3}{\sin 102^\circ} = \frac{x}{\sin 17^\circ}$$

$$\frac{3\sin 17^\circ}{\sin 102^\circ} = \frac{x \sin 102^\circ}{\sin 102^\circ}$$

$$\boxed{x \approx .90 \text{ mi}}$$

$$\frac{3}{\sin 102^\circ} = \frac{y}{\sin 61^\circ}$$

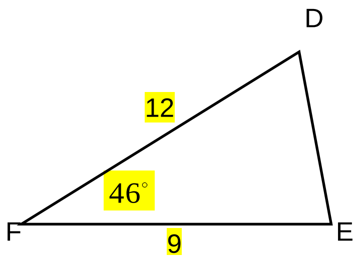
$$\frac{3\sin 61^\circ}{\sin 102^\circ} = \frac{y \sin 102^\circ}{\sin 102^\circ}$$

$$\boxed{y \approx 2.68 \text{ mi}}$$

$$2.68 - .90 = 1.78 \text{ mi}$$

Jimmy is 1.78 miles closer to school than Adolph.

Ex 2: In ΔDEF , $d = 9$ in , $e = 12$ in and $m\angle F = 46^\circ$. Find f to the nearest hundredth.



$$f^2 = d^2 + e^2 - 2de\cos F$$
$$f^2 = 9^2 + 12^2 - 2(9)(12)\cos 46^\circ$$
$$f^2 = 81 + 144 - 216\cos 46^\circ$$
$$f^2 = 225 - 150.05$$
$$f^2 = 74.95$$
$$\sqrt{f^2} = \sqrt{74.95}$$
$$\boxed{f \approx 8.66 \text{ in}}$$

Ex 3: Solve ΔABC if $b = 22$, $c = 30$ and $m\angle C = 30^\circ$. Round all answers to the nearest tenth.

$$\frac{\sin C}{c} = \frac{\sin B}{b}$$
$$\sin C = \frac{c\sin B}{b}$$
$$\sin C = \frac{30\sin 30^\circ}{22}$$
$$C = \sin^{-1}\left(\frac{30\sin 30^\circ}{22}\right)$$
$$C \approx 43.0^\circ$$

Case #1: $m\angle C \approx 43.0^\circ$

$m\angle A = 180 - 30 - 43.0 \approx 107.0^\circ$

$$a = \frac{b\sin A}{\sin B}$$
$$a = \frac{22sin107.0^\circ}{.5}$$
$$a \approx 42.1$$

Case #2: $m\angle C \approx 180 - 43.0 \approx 137.0^\circ$

$m\angle A \approx 180 - 30 - 137.0 \approx 13.0^\circ$

$$a = \frac{b\sin A}{\sin b}$$
$$a = \frac{22\sin 13.0^\circ}{\sin 30^\circ}$$
$$a \approx 9.9$$

Closure:

- When do you apply the Law of Sines?
  Use the Law of Sines when you are given two angles and a side or when you are given two sides and a non-included angle.

- When do you apply the Law of Cosines?
  Use the Law of Cosines when you are given three sides or two sides and the included angle.