CHAPTER # 4– CONGRUENT TRIANGLES

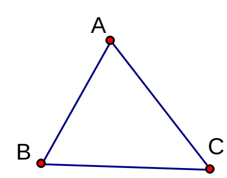
In this chapter we address three **Big IDEAS**:

1) ***Classify triangles by sides and angles***
2) ***Prove that triangles are congruent***
3) ***Use coordinate geometry to investigate triangle relationships***

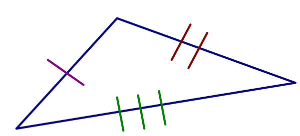
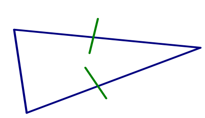
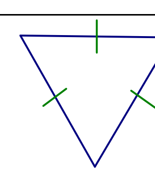
Section:	**4 – 1 Apply Triangle Sum Properties**
Essential Question	How can you find the measure of the third angle of a triangle if you know the measures of the other two angles?

Warm Up:

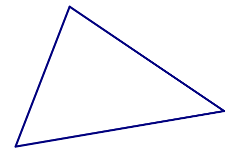
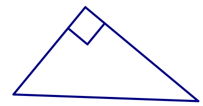
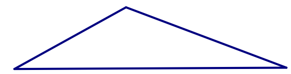
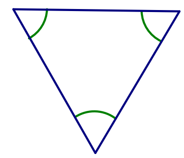
Key Vocab:

Triangle	a polygon with three sides	 ΔABC

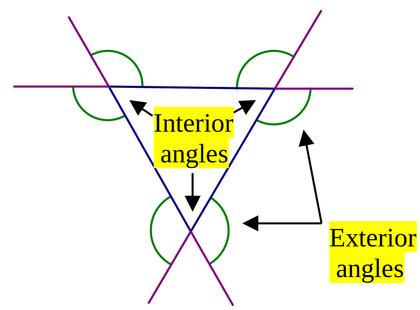
Classifications by Side Lengths:

Scalene Triangle	a triangle with NO congruent sides	
Isosceles Triangle	a triangle with AT LEAST two congruent sides	
Equilateral Triangle	a triangle with three congruent sides	

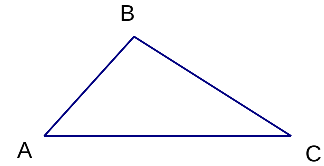
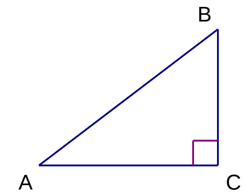
Classifications by Angles Measures:

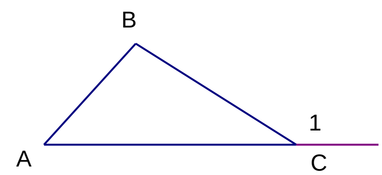
Acute Triangle	a triangle with three acute angles	
Right Triangle	a triangle with one right angle	
Obtuse Triangle	a triangle with one obtuse angle	
Equiangular Triangle	a triangle with three congruent angles	

Additional Vocabulary:

Interior Angle	When the sides of a polygon are extended, the *interior angles* are the original angles.	
Exterior Angle	When the sides of a polygon are extended, the *exterior angles* are the angles that form linear pairs with the interior angles.	
Corollary to a Theorem	A statement that can be proved easily using the theorem to which it is linked.	

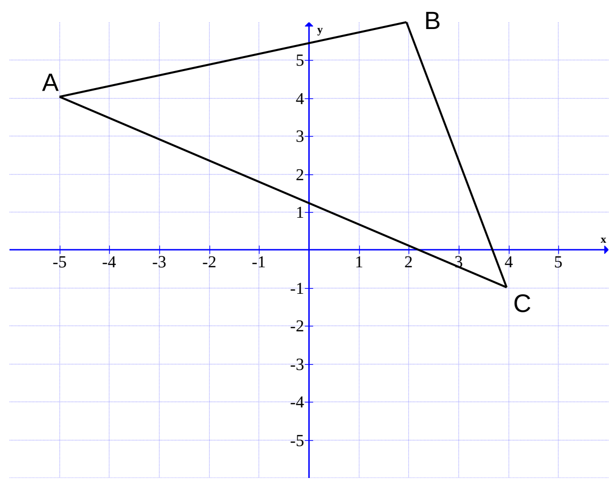
Theorems:

Triangle Sum Theorem	
The sum of the measures of a triangle is $180°$ $m\angle A + m\angle B + m\angle C = 180^{\circ}$	
Corollary to the Triangle Sum Theorem	
The acute angles of a right triangle are complementary $m\angle A + m\angle B = 90^{\circ}$	

Exterior Angle Theorem	
The measure of an exterior angle of a triangle is equal to the sum of the measures of the remote interior angles. $m\angle 1 = m\angle A + m\angle B$	

Show:

Ex 1: Classify ΔABC by its sides and by its angles.



Sides:

$AB = \sqrt{(2 - -5)^2 + (6-4)^2} = \sqrt{49+4} = \sqrt{53}$

$BC = \sqrt{(4-2)^2 + (-1-6)^2} = \sqrt{4+49} = \sqrt{53}$

$AC = \sqrt{(4 - -5)^2 + (-1-4)} = \sqrt{81+25} = \sqrt{106}$

Angles:

$m_{AB} = \dfrac{6-4}{2 - -5} = \dfrac{2}{7}$

$m_{BC} = \dfrac{-1-6}{4-2} = \dfrac{-7}{2}$

$m_{AC} = \dfrac{-1-4}{4 - -5} = \dfrac{-5}{9}$

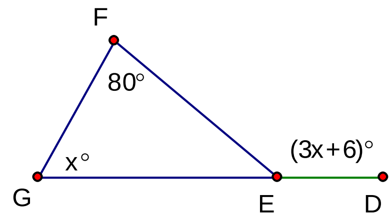
Because $\overline{AB} \cong \overline{BC}$ AND $\overline{AB} \perp \overline{BC}$, ΔABC is an **Isosceles Right Triangle**

Ex 2: Find $m\angle DEF$.

By the Exterior Angle Theorem:

$$3x+6=80+x$$
$$2x=74$$
$$x=37$$

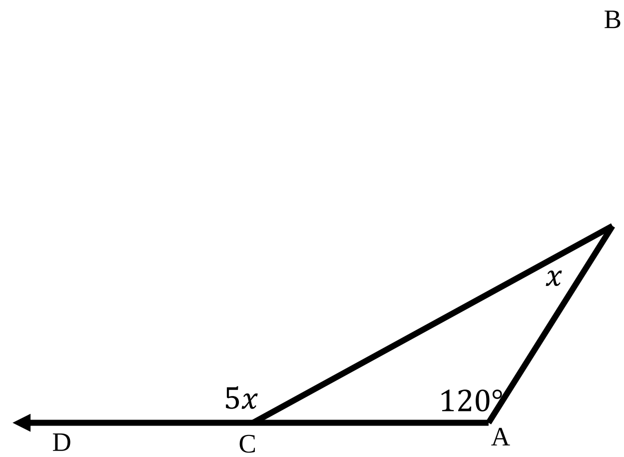
$$3(37)+6=117^{\circ}$$



You Try Ex 3: Find the m∠BCD.

$5x = x + 120$
$4x = 120$
$x = 30$

$m\angle B = 30$

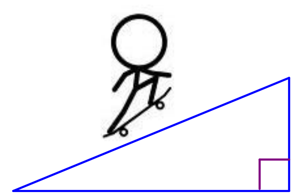


Ex 4: The support for the skateboard ramp shown forms a right triangle. The measure of one acute angle in the triangles is five times the measure of the other. Find the measure of each acute angle.

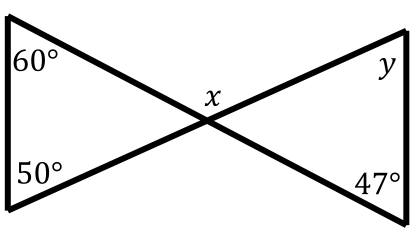
By the Corollary to the Triangle Sum Theorem:

$$x+5x=90$$
$$6x=90$$
$$x=15$$

$$x=15^{\circ}$$
$$5x=75^{\circ}$$



Ex 5: Solve for x and y.



By Exterior Angle Theorem:
$x = 60 + 50 = 110°$
$y = 110 - 47 = 63°$

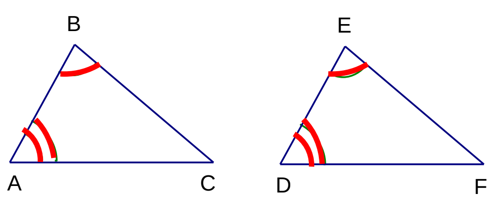
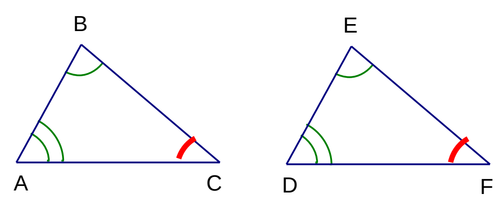
Section:	**4 – 2 Apply Congruence and Triangles**
Essential Question	What are congruent figures?

Warm Up:

Key Vocab:

Congruent Figures	Two or more figures with exactly the same size and shape. All *corresponding parts*, sides and angle, are congruent.
Corresponding Parts	A pair of sides or angles that have the same relative position in two or more congruent figures

Theorems:

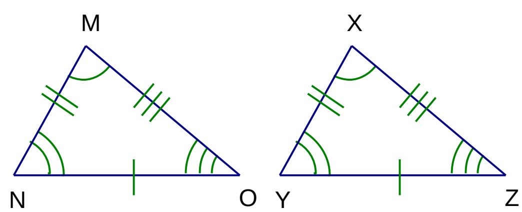
Third Angles Theorem	
If two angles of one triangle are congruent to two angles of another triangle,	**Then** the third angles are also congruent.
$\angle A \cong \angle D$ and $\angle B \cong \angle E$,	$\angle C \cong \angle F$.
	

Properties:

Congruence of Triangles	
Triangle congruence is reflexive, symmetric, and transitive.	
Reflexive	$\Delta ABC \cong \Delta ABC$
Symmetric	If $\Delta ABC \cong \Delta DEF$, then $\Delta DEF \cong \Delta ABC$
Transitive	If $\Delta ABC \cong \Delta DEF$ and $\Delta DEF \cong \Delta JKL$, then $\Delta ABC \cong \Delta JKL$

Show:

Ex 1: Write a congruence statement for the triangles shown. Identify all pairs of congruent corresponding parts

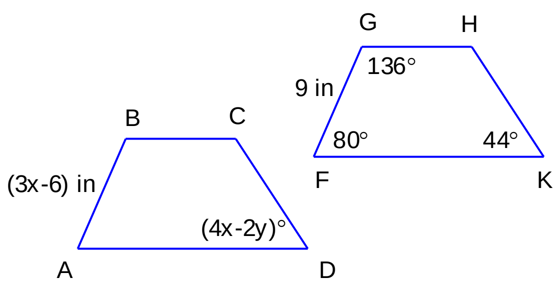


$\Delta NMO \cong \Delta YXZ$

$\overline{NO} \cong \overline{YZ}; \overline{NM} \cong \overline{YX}; \overline{MO} \cong \overline{XZ}$

$\angle MNO \cong \angle XYZ; \angle OMN \cong \angle ZXY;$
$\angle MON \cong \angle XZY$

Ex 2: In the diagram, $ABCD \cong FGHK$



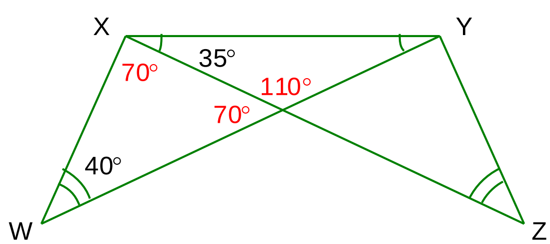
a. Find the value of *x*.

$$3x-6=9$$
$$3x=15$$
$$x=5$$

b. Find the value of *y*.

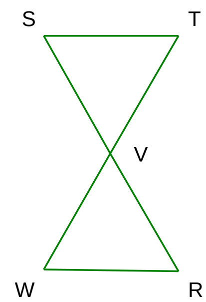
$$4x-2y=44$$
$$4(5)-2y=44$$
$$-2y=24$$
$$y=-12$$

Ex 3: Find $m\angle YXW$.



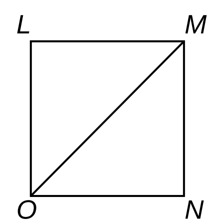
$$180-35-35=110^{\circ}$$
$$180-110=70^{\circ}$$
$$180-40-70=70^{\circ}$$
$$m\angle YXW = 70+35=105^{\circ}$$

Ex 4: Given: $\overline{SV} \cong \overline{RV}, \overline{TV} \cong \overline{WV}$,
$\overline{ST} \cong \overline{RW}, \angle T \cong \angle W$
Prove: $\Delta STV \cong \Delta RWV$



Statements	Reasons
1. $\overline{SV} \cong \overline{RV}, \overline{TV} \cong \overline{WV}$, $\overline{ST} \cong \overline{RW}, \angle T \cong \angle W$	1. Given
2. $\angle SVT \cong \angle RVW$	2. Vert. $\angle$'s Thm
3. $\angle S \cong \angle R$	3. Third $\angle$'s Thm
4. $\Delta STV \cong \Delta RWV$	4. Def. of $\cong \Delta$'s

Ex 5: Given: Quad $LMNO$ is a square
$\overline{MO}$ bisects $\angle LMN$ and $\angle NOL$
Prove: $\Delta OLM \cong \Delta ONM$



Statements	Reasons
1. Quad $LMNO$ is a square $\overline{MO}$ bisects $\angle LMN$ and $\angle NOL$	1. Given
2. $\overline{OL} \cong \overline{ON} \cong \overline{NM} \cong \overline{ML}$	2. Definition of a Square
3. $\angle LOM \cong \angle MON$ $\angle LMO \cong \angle OMN$	3. Definition of an angle bisector
4. $\overline{OM} \cong \overline{OM}$	4. Reflexive Prop.
5. $\Delta OLM \cong \Delta ONM$	5. Def. of $\cong \Delta$'s

Closure:

- How do you know two figures are congruent?

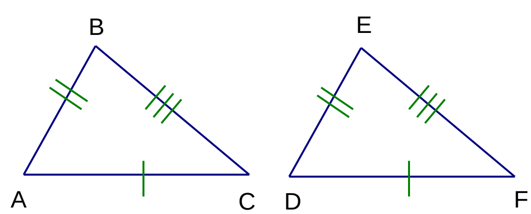
  ALL corresponding sides and ALL corresponding angles must be congruent.

Section:	**4 – 4 Prove Triangles Congruent by SSS**
Essential Question	How can you use side lengths to prove triangles congruent?

Warm Up:

Postulate

Side-side-side (SSS) Congruence Postulate	
If three sides of one triangle are congruent to three sides of a second triangle,	**then** the two triangles are congruent.
$\overline{AB} \cong \overline{DE}, \overline{BC} \cong \overline{EF}$, and $\overline{AC} \cong \overline{DF}$	$\Delta ABC \cong \Delta DEF$



Show:

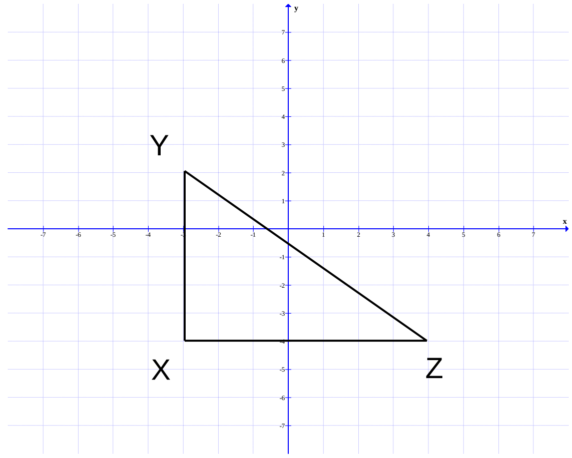
Ex1: **Multiple Choice:** Which are the coordinates of the vertices of a triangle congruent to ΔXYZ ?

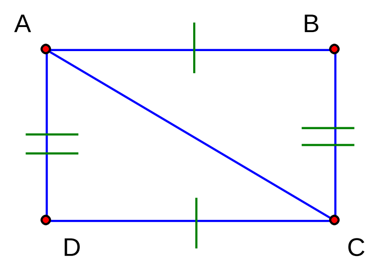
A. $(6,2),(0,-6),(6,-5)$

B. $(5,1),(-1,-6),(5,-6)$

C. $(4,0),(-1,-7),(4,-7)$

D. $(3,-1),(-3,-7),(3,-8)$





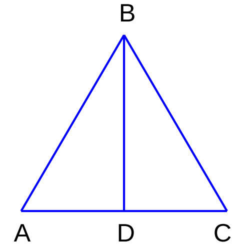
Ex2: Given: Diagram
Prove $\Delta ABD \cong \Delta CDB$

Statements	Reasons
1. $\overline{AB} \cong \overline{CD}; \overline{AD} \cong \overline{BC}$	1. Given
2. $\overline{AC} \cong \overline{AC}$	2. Reflexive Prop.
3. $\Delta ABD \cong \Delta CDB$	3. SSS $\cong$ Post.

Ex3: Given: D is the midpoint of $\overline{AC}$

$\overline{AB} \cong \overline{BC}$

Prove: $\Delta ABD \cong \Delta CBD$



Statements	Reasons
1. D is the midpoint of $\overline{AC}$; $\overline{AB} \cong \overline{BC}$	a. Given
2. $\overline{BD} \cong \overline{BD}$	2. Reflexive Property
3. $\overline{AD} \cong \overline{DC}$	3. Definition of a midpoint
4. $\Delta ABD \cong \Delta CBD$	4. SSS $\cong$ Postulate

Closure:

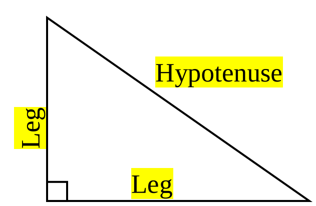
- Can you use side lengths to prove quadrilaterals congruent?

No, SSS can *only* be applied to triangles. Four sides can be arranged in different orders to create different quadrilaterals, whereas three sides will create a unique triangle.

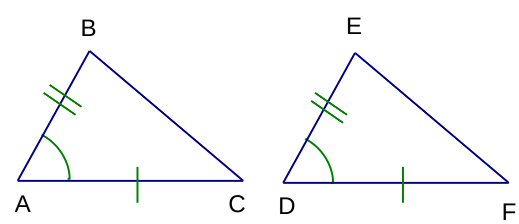
Section:	**4 – 5 Prove Triangles Congruent by SAS and HL**
Essential Question	How can you use two sides and an angle to prove triangles congruent?

Warm Up:

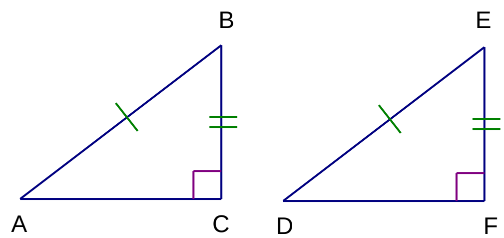
Key Vocab:

Legs (of a Right Triangle)	In a right triangle, the sides adjacent to the right angle.	
Hypotenuse	In a right triangle, the side opposite the right angle Always the longest side of a right triangle	

Side-Angle-Side (SAS) Congruence Postulate	
If two sides and the included angle of one triangle are congruent to two sides and the *included* angle of a second triangle,	**then** the two triangles are congruent.
$\overline{AB} \cong \overline{DE}, \angle A \cong \angle D,$ and $\overline{AC} \cong \overline{DF}$	$\Delta ABC \cong \Delta DEF$

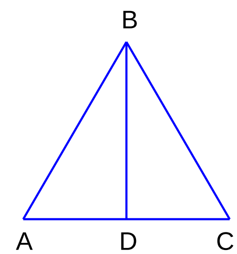


Hypotenuse-Leg (HL) Theorem	
If the hypotenuse and a leg of a right triangle are congruent to the hypotenuse and a leg of a second right triangle,	**then** the two triangles are congruent.
$\overline{AB} \cong \overline{DE}, \overline{BC} \cong \overline{EF}$, and ΔABC and ΔDEF **are right triangles**	$\Delta ABC \cong \Delta DEF$



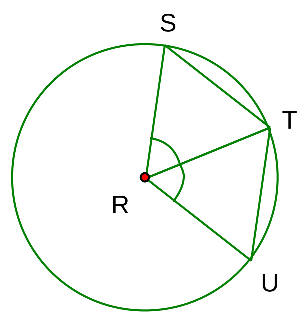
Show:

Ex 1: If you know that $\overline{AB} \cong \overline{CB}$ and $\angle ABD \cong \angle CBD$, what postulate or theorem can you use to conclude that $\Delta ABD \cong \Delta CBD$?



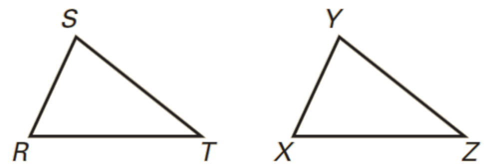
The SAS Post.

Ex 2: In the diagram R is the center of the circle. If $\angle SRT \cong \angle URT$, what can you conclude about ΔSRT and ΔURT?



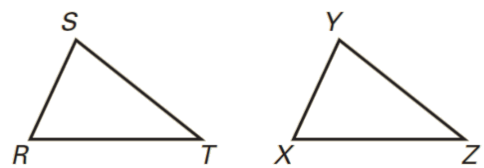
They are congruent by SAS Post.

Ex 3: State the third congruence that would allow you to prove $\Delta RST \cong \Delta XYZ$ by the SAS Congruence Postulate.



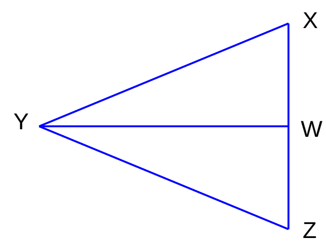
a. $\overline{ST} \cong \overline{YZ}, \overline{RS} \cong \overline{XY}$

$\angle S \cong \angle Y$



b. $\angle T \cong \angle Z, \overline{RT} \cong \overline{XZ}$

$\overline{ST} \cong \overline{YZ}$



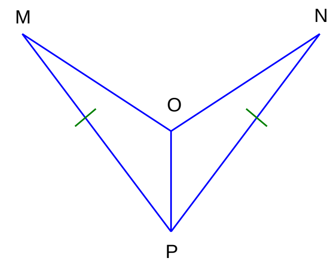
Ex 4: Given: $\overline{YW} \perp \overline{XZ}; \overline{XY} \cong \overline{ZY}$

Prove: $\Delta XYW \cong \Delta ZYW$

Statements	Reasons
1. $\overline{YW} \perp \overline{XZ}; \overline{XY} \cong \overline{ZY}$	1. Given
2. $\angle XWY$ and $\angle ZWY$ are rt. $\angle$'s	2. $\perp$ lines form 4 rt. $\angle$'s
3. $\Delta XYW \cong \Delta ZYW$ are rt. Δ's	3. Def. of rt. Δ
4. $\overline{YW} \cong \overline{YW}$	4. Reflexive Prop.
5. $\Delta XYW \cong \Delta ZYW$	5. HL Thm.

Ex 5: Given: $\overline{MP} \cong \overline{NP}; \overline{OP}$ bisects $\angle MPN$

Prove: $\Delta MOP \cong \Delta NOP$



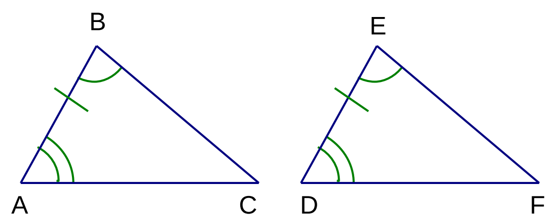
Statements	Reasons
1. $\overline{MP} \cong \overline{NP}; \overline{OP}$ bisects $\angle MPN$	1. Given
2. $\angle MPO \cong \angle NPO$	2. Def. of $\angle$ bis.
3. $\overline{OP} \cong \overline{OP}$	3. Reflexive Prop
4. $\Delta MOP \cong \Delta NOP$	4. SAS Post.

Section:	**4 – 6 Prove Triangles Congruent by ASA and AAS**
Essential Question	If one side of a triangle is congruent to one side of another, what do you need to know about the angles to prove the triangles are congruent?

Warm Up:

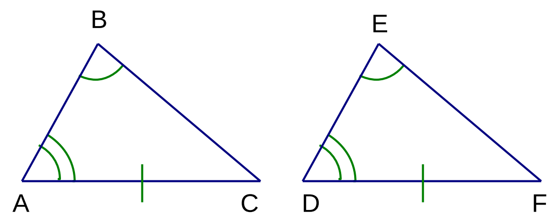
Postulates:

Angle-Side-Angle (ASA) Congruence Postulate	
If two angles and the *included* side of one triangle are congruent to two angles and the *included* side of a second triangle,	**then** the two triangles are congruent.
$\angle A \cong \angle D, \overline{AB} \cong \overline{DE}$, and $\angle B \cong \angle E$	$\Delta ABC \cong \Delta DEF$



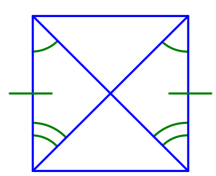
Theorems:

Angle-Angle-Side (AAS) Congruence Theorem	
If two angles and a *non-included* side of one triangle are congruent to two angles and a *non-included* side of a second triangle,	**Then** the two triangles are congruent.
$\angle B \cong \angle E, \angle A \cong \angle D,$ and $\overline{AC} \cong \overline{DF}$	$\Delta ABC \cong \Delta DEF$



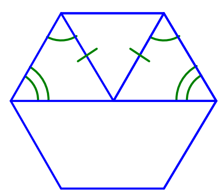
Ex 1: Can the triangles be proven congruent with the information given in the diagram? If so, state the postulate or theorem you would use.

a.



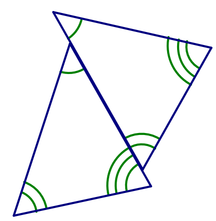
ASA Post

b.



AAS Theorem

c.



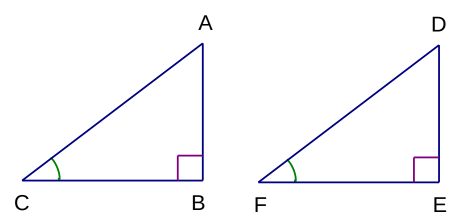
Cannot be proven congruent

Ex 2: Write a two-column proof.

Given: $\overline{AB} \perp \overline{BC}; \overline{DE} \perp \overline{EF}$

$\overline{AC} \cong \overline{DF}; \angle C \cong \angle F$

Prove: $\Delta ABC \cong \Delta DEF$



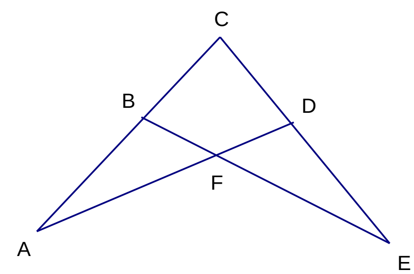
Statements	Reasons
1. $\overline{AB} \perp \overline{BC}; \overline{DE} \perp \overline{EF}$	1. Given
2. $\angle B$ is a rt. $\angle$; $\angle E$ is a rt. $\angle$	2. Def. of $\perp$ lines
3. $\angle B \cong \angle E$	3. Rt. $\angle \cong$ Thm.
4. $\overline{AC} \cong \overline{DF}; \angle C \cong \angle F$	4. Given
5. $\Delta ABC \cong \Delta DEF$	5. AAS $\cong$ Post.

Ex 3: Write a proof:

Given: $\angle CBF \cong \angle CDF$

$\overline{BF} \cong \overline{FD}$

Prove: $\Delta ABF \cong \Delta EDF$



Statements	Reasons
1. $\angle CBF \cong \angle CDF$ $\overline{BF} \cong \overline{FD}$	1. Given
2. $\angle CBF$ and $\angle ABF$ are supplementary $\angle CDF$ and $\angle EDF$ are supplementary	2. Linear Pair Postulate
3. $\angle ABF \cong \angle EDF$	3. Congruent Supplements Theorem
4. $\angle BFA \cong \angle DFE$	4. Vertical Angles Theorem
5. $\Delta ABF \cong \angle EDF$	5. ASA Postulate

Closure:

- What are the FIVE ways to prove that two triangles are congruent?
  1. SSS Congruence Postulate
  2. SAS Congruence Postulate
  3. ASA Congruence Postulate
  4. AAS Congruence Theorem
  5. HL Congruence Theorem

Section:	**4 – 7 Use Congruent Triangles**
Essential Question	How can you use congruent triangles to prove angles or sides congruent?

Warm Up:

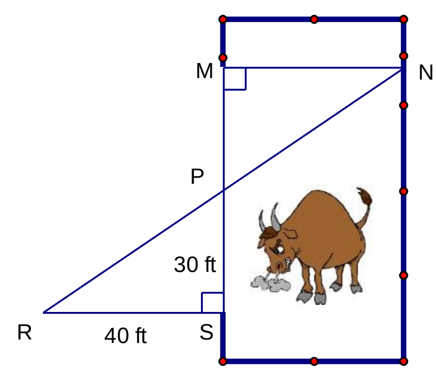
Key Vocab:

CPCTC	Corresponding Parts of Congruent Triangles are Congruent

Show:

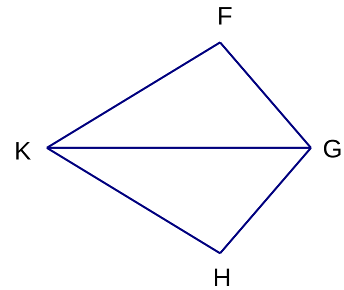
Ex 1: If P is the midpoint of $\overline{MS}$, how wide is the bull's pasture?

40 feet



Ex 2: Write a two-column proof.

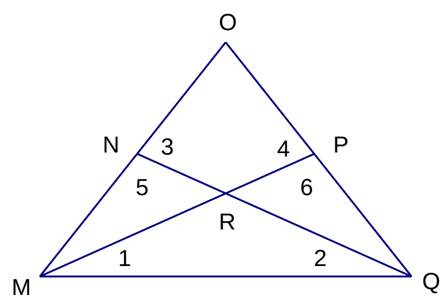
Given: $\overline{GK}$ bisects $\angle FGH$ and $\angle FKH$
Prove: $\overline{FK} \cong \overline{HK}$



Statements	Reasons
1. $\overline{GK}$ bisects $\angle FGH$ and $\angle FKH$	1. Given
2. $\angle FGK \cong \angle HGK; \angle FKG \cong \angle HKG$	2. Def. of $\angle$ bis.
3. $\overline{GK} \cong \overline{GK}$	3. Reflexive Prop.
4. $\Delta FGK \cong \Delta HGK$	4. ASA Post.
5. $\overline{FK} \cong \overline{HK}$	5. CPCTC

Ex 3: Write a flow proof:

Given: $\angle 1 \cong \angle 2; \angle 3 \cong \angle 4$
Prove: $\Delta MNR \cong \Delta QPR$

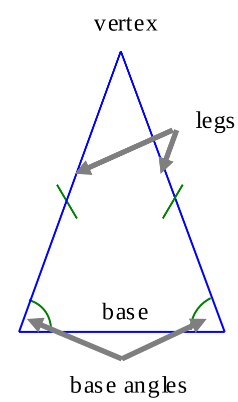


Statements	Reasons
1. $\angle 1 \cong \angle 2; \angle 3 \cong \angle 4$	1. Given
2. $\overline{MQ} \cong \overline{MQ}$	2. Reflexive Property
3. $\angle 5 \cong \angle 6$	3. Congruent Supplements Theorem
4. ΔMNQ≅ΔQPN	4. AAS
5. ∠NRM≅∠PRQ	5. Vertical Angles Congruence Theorme
6. $\overline{MN} \cong \overline{PQ}$	6. CPCTC
7. $\Delta MNR \cong \Delta QPR$	7. AAS

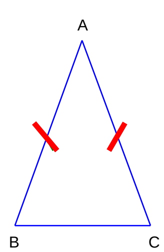
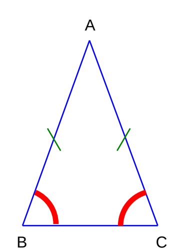
Section:	**4 – 8 Use Isosceles and Equilateral Triangles**
Essential Question	How are the sides and angles of a triangle related if there are two or more congruent sides or angles?

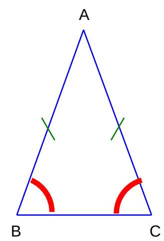
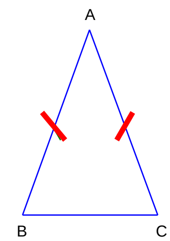
Warm Up:

Key Vocab:

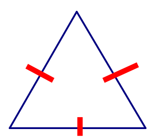
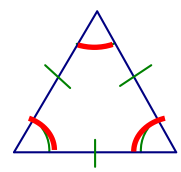
Components of an Isosceles Triangle		
Legs	The congruent sides	
Vertex Angle	The angle formed by the legs	
Base	The third side (the side that is NOT a leg)	
Base Angle	The two angles that are adjacent to the base	

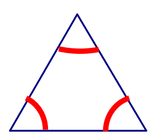
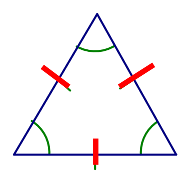
Theorems:

Base Angles Theorem (Isosceles Triangle Theorem)	
If two sides of a triangle are congruent,	**then** the angles opposite them are congruent.
$\overline{AB} \cong \overline{AC}$	$\angle B \cong \angle C$
	

Base Angles Theorem Converse (Isosceles Triangle Theorem Converse)	
If two angles of a triangle are congruent,	**then** the sides opposite them are congruent.
$\angle B \cong \angle C$	$\overline{AB} \cong \overline{AC}$
	

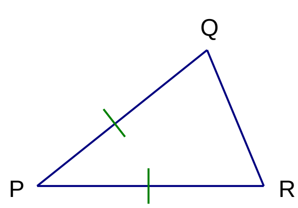
Corollaries:

If a triangle is equilateral,	**then** it is equiangular.
	

If it is equiangular.	**then** a triangle is equilateral,
	

Show:

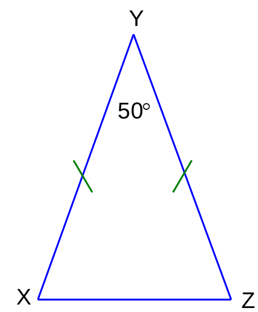
Ex 1: In $\Delta PQR, \overline{PQ} \cong \overline{PR}$. Name two congruent angles.



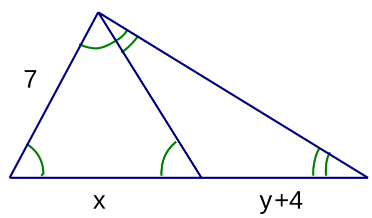
$\angle Q \cong \angle R$

Ex 2: Find the measure of $\angle X$ and $\angle Z$.

$65^{\circ}, 65^{\circ}$

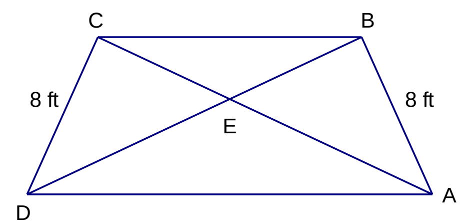


Ex 3: Find the values of x and y in the diagram.



$x = 7, y = 3$

Ex 4: Diagonal braces $\overline{AC}$ and $\overline{BD}$ are used to reinforce a signboard that advertises fresh eggs and produce at a roadside stand. Each brace is 14 feet long.



a. What congruent postulate can you use to prove that $\Delta ABC \cong \Delta DCB$?

SSS Post.

b. Explain why ΔBEC is isosceles.

$\angle DBC \cong \angle ACB$, since **CPCTC**. $\overline{BE} \cong \overline{CE}$ by the **Conv. of the Base** $\angle$'s **Thm**. And this implies that ΔBEC is isosceles.

c. What triangles would you use to show that ΔAED is isosceles?

ΔABD and ΔDCA