CHAPTER 10 – PROPERTIES OF CIRCLES

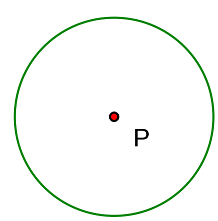
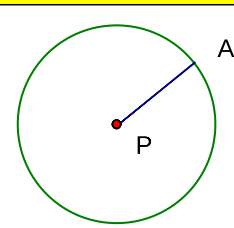
In this chapter we address
Big IDEAS:

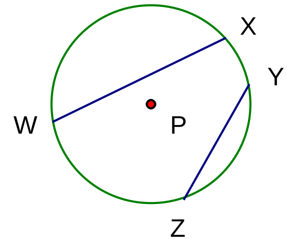
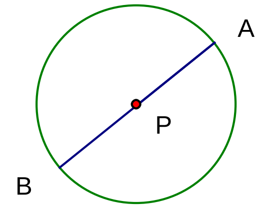
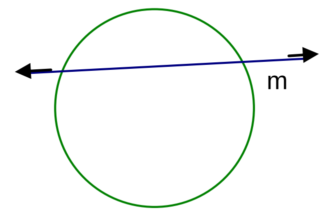
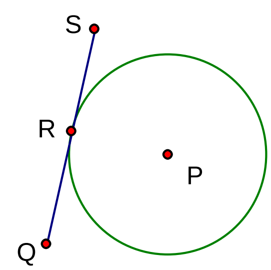
1) ***Using properties of segments that intersect circles***
2) ***Applying angle relationships in circles***
3) ***Using circles in the coordinate plane***

Section:	**10 – 1 Use Properties of Tangents**
Essential Question	**How can you verify that a segment is tangent to a circle?**

Warm Up:

Key Vocab:

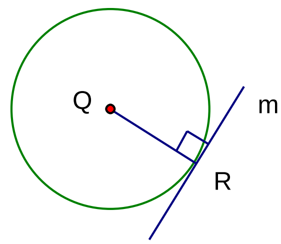
Circle	The set of all points in a plane that are equidistant from a given point called the *center* of the circle.	 Point *P* is the *center* of ⊙*P*
Center	The point from which all points of the circle are equidistant	
Radius	A segment whose endpoints are the center of the circle and a point on the circle. The distance from the center of a circle to any point on the circle. Plural: radii	 $\overline{PA}$ is a *radius*

Chord	A segment whose endpoints are on the circle.	 $\overline{WX}$ and $\overline{YZ}$ are *chords*.
Diameter	A *chord* that passes through the center of the circle. The distance across a circle through its center.	 $\overline{AB}$ is a *diameter*
Secant	A line that intersects a circle in two points.	 Line *m* is a *secant.*
Tangent	A line in the plane of a circle that intersects the circle in exactly one point, the *point of tangency*.	 $\overline{QS}$ is tangent to $\odot P$ at *point of tangency R.*

Theorems:

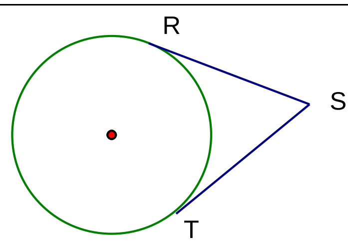
In a plane,

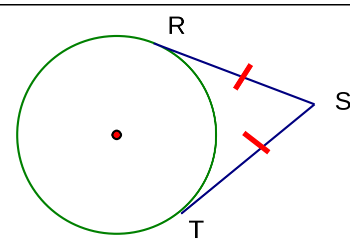
a line is tangent to a circle **IFF** the line is perpendicular to a radius of the circle at its endpoint on the circle



If line m is tangent to $\odot Q$,	**Then** $m \perp \overline{QR}$
If $m \perp \overline{QR}$	**Then** line m is tangent to $\odot Q$.

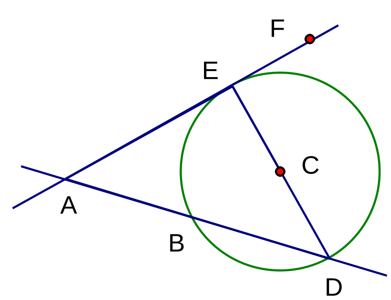
If	**Then**
Tangent segments are drawn from a common external point,	They are congruent.
$\overline{RS}$ and $\overline{TS}$ are tangents,	$\overline{RS} \cong \overline{TS}$





Show:

Ex 1: Tell whether the line or segment is best described as a radius chord, diameter, secant, or tangent of $\odot C$.



a.) $\overline{DC}$ radius

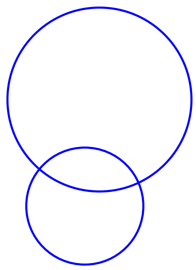
b.) $\overline{BD}$ chord

c.) $\overline{DE}$ diameter

d.) $\overleftrightarrow{AE}$ tangent

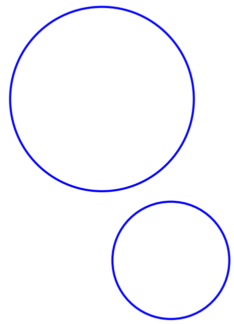
Ex 2: Tell how many common tangents the circles have and draw them.

a.)



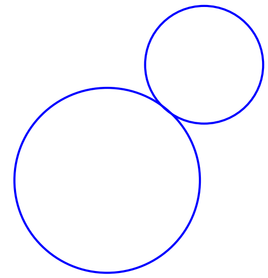
2 external tangents

b.)



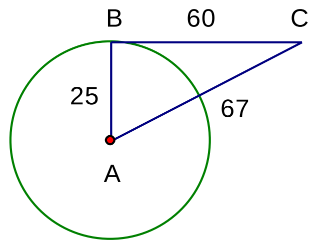
4 common tangents – 2 external, 2 internal

c.)



3 common tangents – 2 external, 1 internal

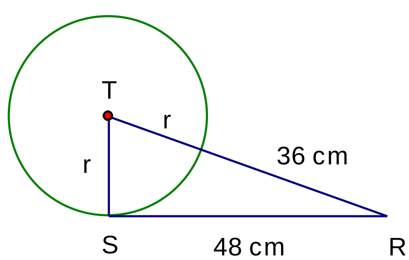
Ex 3: In the diagram, $\overline{AB}$ is a radius of $\odot A$. Is $\overline{BC}$ tangent to $\odot A$? Explain.



No, $25^2 + 60^2 = 4225$, and $67^2 = 4489$

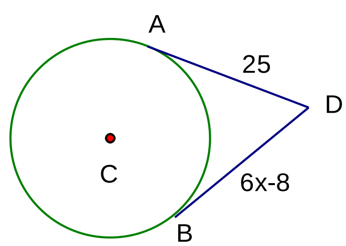
$\therefore 25^2 + 60^2 \neq 67^2$

Ex 4: In the diagram, S is a point of tangency. Find the radius r of $\odot T$.



$$(r+36)^2 = r^2 + 48^2$$
$$r^2 + 72r + 1296 = r^2 + 2304$$
$$72r = 1018$$
$$r = 14 \text{ cm}$$

Ex 5: In $\odot C$, $\overline{DA}$ is tangent at A and $\overline{DB}$ is tangent at B. Find x.

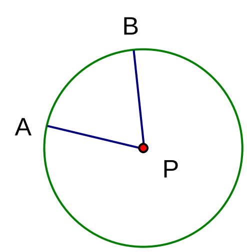
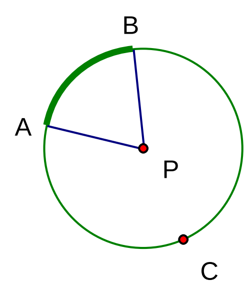


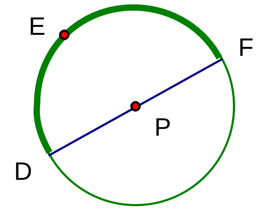
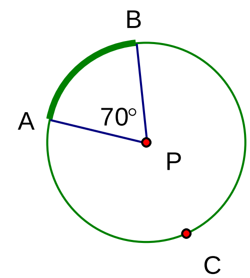
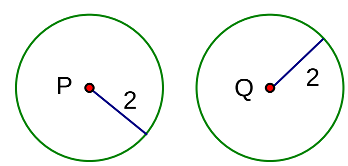
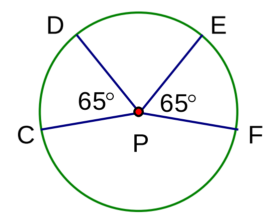
$$25 = 6x - 8$$
$$33 = 6x$$
$$5.5 = x$$

Section:	**10 – 2 Find Arc Measures**
Essential Question	How do you find the measure of an arc of a circle?

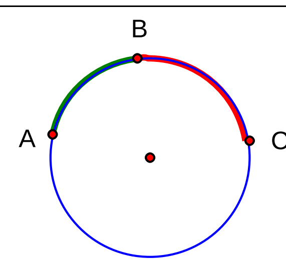
Warm Up:

Key Vocab:

Central Angle	An angle whose vertex is the center of the circle.	 $\angle APB$ is a *central angle* of $\odot P$
Arc	A portion of the circumference of the circle.	 AB is a minor arc ACB is a major arc
Minor Arc	Part of a circle that measures less than 180°	
Major Arc	Part of a circle that measures greater than 180°	

Semicircle	An arc with endpoints that are the endpoints of a diameter of a circle. The measure of a semicircle is 180°	 *DEF* is a *semicircle.*
Measure of an Arc	The measure of the arc's central angle	 $mAB = 70°$ $mACB = 360° - 70° = 290°$
Congruent Circles	Circles that have congruent radii	 $\odot P \cong \odot Q$
Congruent Arcs	Two arcs that have the same measure and are arcs of the same circle or of congruent circles.	 $CD \cong EF$

Postulates:

Arc Addition Postulate		
The measure of an arc formed by two adjacent arcs is the sum of the measures of the two arcs.		
The sum of the parts equals the whole		
If *B* is between *A* and *C*,	**then** $mABC = mAB + mBC$	
If $mABC = mAB + mBC$,	**then** *B* is between *A* and *C*.	

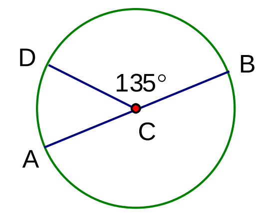
Show:

Ex 1: Find the measure of each arc of $\odot C$, where $\overline{AB}$ is a diameter

a.) $m\overset{\frown}{DB} = 135°$

b.) $m\overset{\frown}{DAB} = 225°$

c.) $m\overset{\frown}{ADB} = 180°$



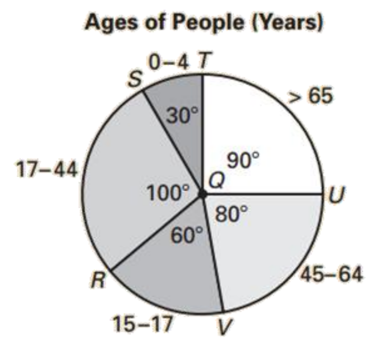
Ex 2: A result of a survey about the ages of people in a town is shown. Find the indicated arc measures.

a.) $m\overset{\frown}{RU} = 140°$

b.) $m\overset{\frown}{RST} = 130°$

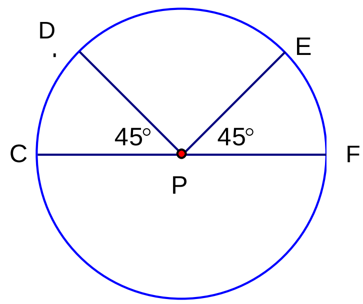
c.) $m\overset{\frown}{RVT} = 230°$

d.) $m\overset{\frown}{UST} = 270°$



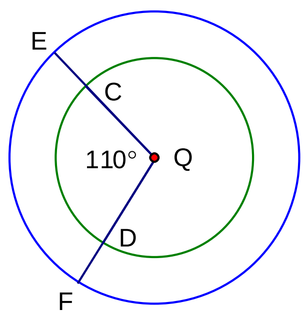
Ex 3: Tell whether arcs CD and EF are congruent. Explain why or why not.

a.)



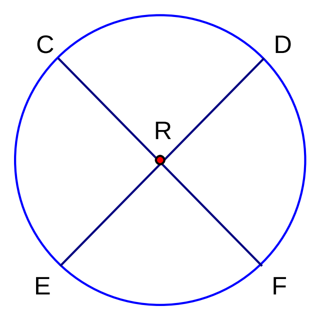
$\overset{\frown}{CD} \cong \overset{\frown}{EF}$; they are in the same circle and $m\overset{\frown}{CD} \cong m\overset{\frown}{EF}$.

b.)



$\overset{\frown}{CD}$ and $\overset{\frown}{EF}$ have the same measure, but they are not congruent because they are arcs of circles that are not congruent.

c.)



$\overset{\frown}{CD} \cong \overset{\frown}{EF}$ because they are in the same circle and vertical angles $\angle CRD$ and $\angle ERF$ are congruent.

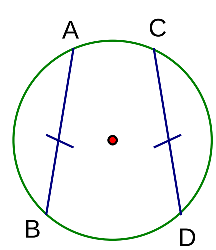
Section:	**10 – 3 Apply Properties of Chords**
Essential Question	How can you tell if two chords in a circle are congruent?

Warm Up:

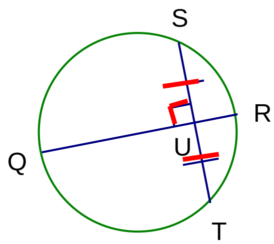
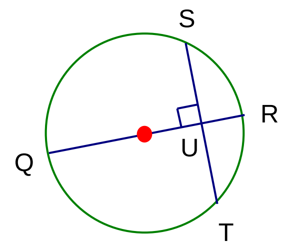
Theorems:

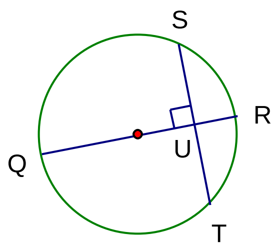
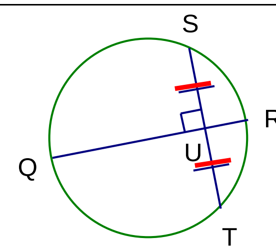
In the same circle or in congruent circles,

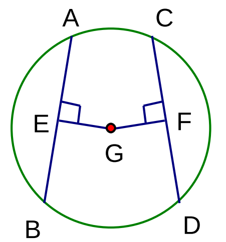
two minor arcs are congruent **IFF** their corresponding chords are congruent.



If $AB \cong CD$,	**Then** $\overline{AB} \cong \overline{CD}$
If $\overline{AB} \cong \overline{CD}$	**Then** $AB \cong CD$.

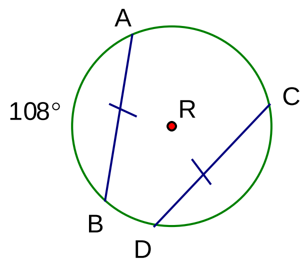
If	**Then**
one chord is a perpendicular bisector of another chord,	the first chord is a diameter.
$\overline{QR}$ is perp. bis. of $\overline{ST}$,	$\overline{QR}$ is a diameter
	

If	**Then**
a diameter of a circle is perpendicular to a chord,	the diameter bisects the chord **AND** its arc.
$\overline{QR}$ is a diameter and $\overline{QR} \perp \overline{ST}$,	$\overline{SU} \cong \overline{TU}$ and $\overset{\frown}{SR} \cong \overset{\frown}{RT}$.
	

In the same circle or in congruent circles, two chords are congruent **IFF** they are equidistant from the center.	
	
If $\overline{AB} \cong \overline{CD}$,	**Then** $EG = FG$
If $EG = FG$	**Then** $\overline{AB} \cong \overline{CD}$.

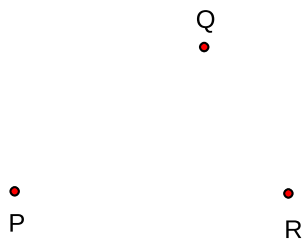
Show:

Ex 1: In $\odot R$, $\overline{AB} \cong \overline{CD}$ and $mAB = 108°$. Find mCD.



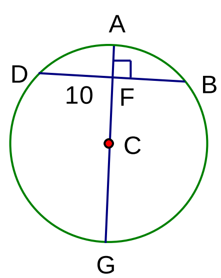
$mCD = 108°$

Ex 2: Three props are place on a stage at points *P*, *Q*, and *R* as shown. Describe how to find the location of a table so it is the same distance from each prop.



Draw the perpendicular bisectors of any two of $\overline{PQ}, \overline{QR},$ and $\overline{PR}$. The intersection of the two perpendicular bisectors is the desired location.

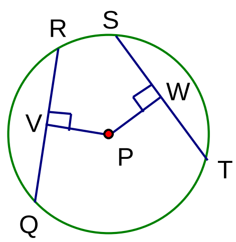
Ex 3: Use the diagram of $\odot C$ to find the length of $\overline{BF}$. Tell what theorem your used.



$BF = 10$

If a diameter of a circle is perpendicular to a chord, then the diameter bisects the chord and its arc.

Ex 4: In the diagram of $\odot P$, $PV = PW$, $QR = 2x + 6$, and $ST = 3x - 1$. Find *QR*.

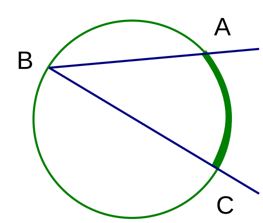
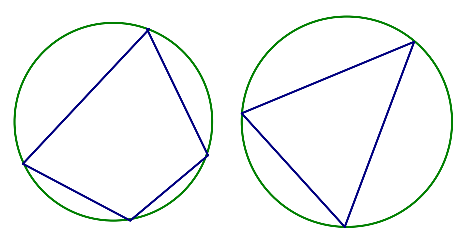


$2x + 6 = 3x - 1$	$QR = 2(7) + 6$
$7 = x$	$QR = 20$

Section:	**10 – 4 Use Inscribed Angles and Polygons**
Essential Question	How do you find the measure of an inscribed angle?

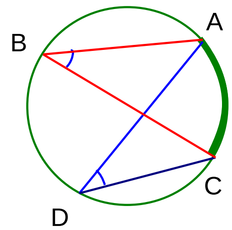
Warm Up:

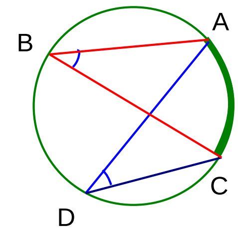
Key Vocab:

Inscribed Angle	An angle whose vertex is on a circle and whose sides contain chords of the circle. The measure of an inscribed angle is one half the measure of its intercepted arc.	 $\angle ABC$ is an *inscribed angle.* AC is its *intercepted arc.* $m\angle ABC = \frac{1}{2} mAC$
Intercepted Arc	The arc that lies in the interior of an inscribed angle and has endpoints on the angle.	
Inscribed Polygon	A polygon whose vertices all lie on a circle.	 The quadrilateral and the triangle are *inscribed* in the circles. The circles are *circumscribed* about the quadrilateral and the triangle.
Circumscribed Circle	The circle that contains the vertices of an inscribed polygon.	

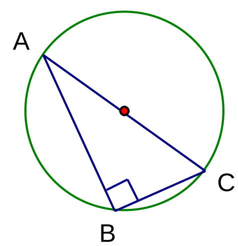
Theorems:

If	**Then**
two inscribed angles of a circle intercept the same arc,	the angles are congruent.
Inscribed angles $\angle ABC$ and $\angle ADC$ both intercept AC,	$\angle ABC \cong \angle ADC$



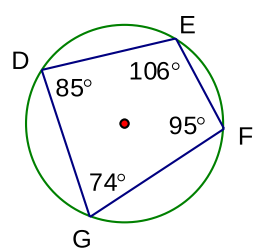


A right triangle can be inscribed in a circle **IFF** the hypotenuse is a diameter of the circle.	



If	**Then**
in ΔABC, $m\angle B = 90°$,	$\overline{AC}$ is a diameter of the circle
$\overline{AC}$ is a diameter of the circle	in ΔABC, $m\angle B = 90°$.

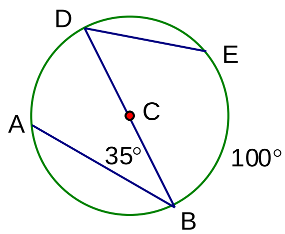
A quadrilateral can be inscribed in a circle **IFF** its opposite angles are supplementary.	



If	**Then**
$D,E,F,$ and G lie on the circle,	$m\angle D + m\angle F = 180 = m\angle G + m\angle E$
$m\angle D + m\angle F = 180 = m\angle G + m\angle E$	$D,E,F,$ and G lie on the circle.

Show:

Ex 1: Find the indicated measure in $\odot C$.

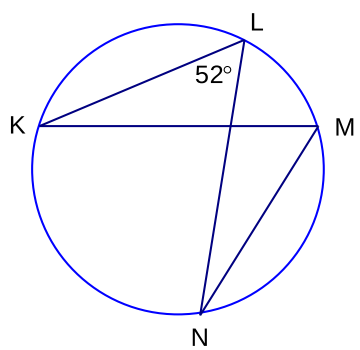


a.) $m\angle D = \frac{1}{2} \cdot 100 = 50°$

b.) $m\overset{\frown}{AB} = 110°$

$m\angle ACB = m\overset{\frown}{AB} = 180 - 35 - 35 = 110°$

Ex 2: Find $m\overset{\frown}{KN}$ and $m\angle KMN$. What do you notice about $\angle KMN$ and $\angle KLN$?

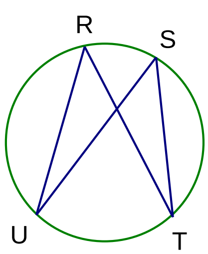


$m\overset{\frown}{KN} = 2(52) = 104°$

$m\angle KMN = 52°$

$\angle KMN \cong \angle KLN$

Ex 3: Name two pair of congruent angles in the figure.



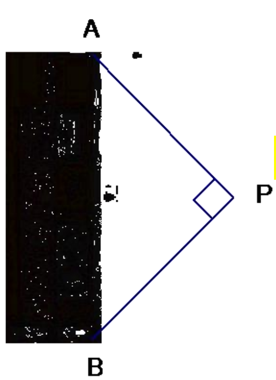
A.) $\angle R \cong \angle S$ and $\angle U \cong \angle T$

B.) $\angle R \cong \angle T$ and $\angle U \cong \angle S$

C.) $\angle R \cong \angle U$ and $\angle S \cong \angle T$

D.) $\angle R \cong \angle T$ and $\angle R \cong \angle S$

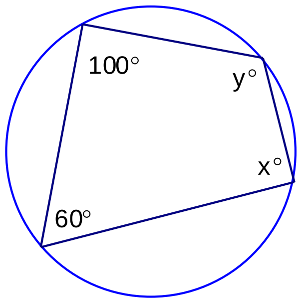
Ex 4: A graphic design software program was used in a home improvement store to design kitchen cabinets. The designer showed a wall of cabinets with a $90°$ viewing angle at *P*. From what other positions would the cabinets fill a $90°$ viewing window?



From any position on a semicircle that has $\overline{AB}$ as a diameter.

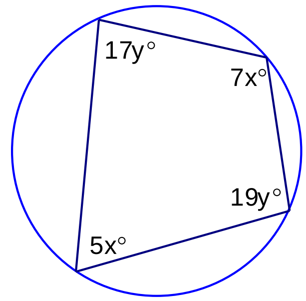
Ex 5: Find the value of each variable.

a.)



$$100 + x = 180 \qquad 60 + y = 180$$
$$x = 80^{\circ} \qquad y = 120^{\circ}$$

b.)



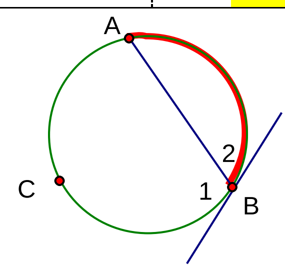
$$5x + 7x = 180 \qquad 17y + 19y = 180$$
$$12x = 180 \qquad 36y = 180$$
$$x = 15 \qquad y = 5$$

Section:	**10 – 5 Apply Other Angle Relationships in Circles**
Essential Question	How do you find the measure of an angle formed by two chords that intersect inside a circle?

Warm Up:

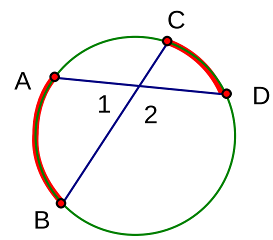
Theorems:

If	**Then**
a tangent and a chord intersect at a point on a circle,	the measure of each angle formed is one half the measure of its intercepted arc.
	$m\angle 1 = \frac{1}{2} m ACB$ and $m\angle 2 = \frac{1}{2} m AB$



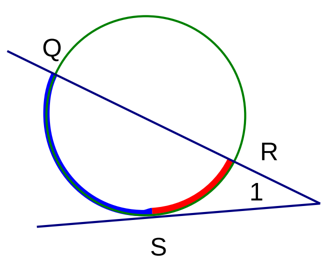
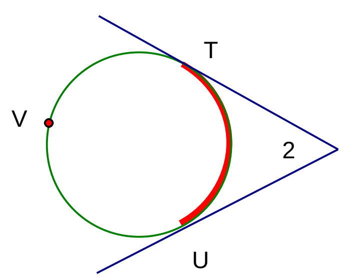
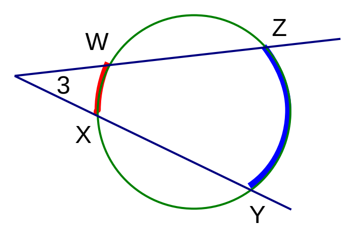
Angles Inside the Circle Theorem

If	Then
two chords intersect inside a circle,	then the measure of each angle is one half the sum of the measures of the arcs intercepted by the angle and its vertical angle..
	$m\angle 1=\frac{1}{2}\left(mAB+mCD\right)$ and $m\angle 2=\frac{1}{2}\left(mAC+mBD\right)$



Angles Outside the Circle Theorem

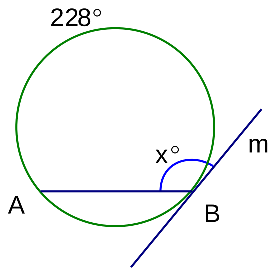
If	Then
a tangent and a secant, two tangents, or two secants intersect outside a circle,,	the measure of the angle formed is one half the difference of the measures of the intercepted arcs

a tangent and a secant	two tangents	two secants
		
$m\angle 1=\frac{1}{2}\left(mQS-mRS\right)$	$m\angle 2=\frac{1}{2}\left(mTVU-mTU\right)$	$m\angle 3=\frac{1}{2}\left(mZY-mWX\right)$

Show:

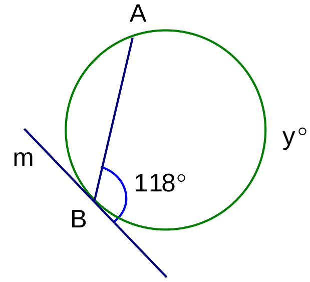
Ex 1: Line *m* is tangent to the circle. Find the indicated variable.

a.)



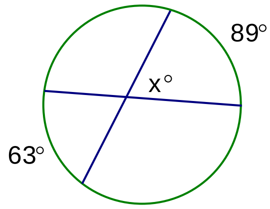
$$x=\frac{1}{2}(228)=114^{\circ}$$

b.)



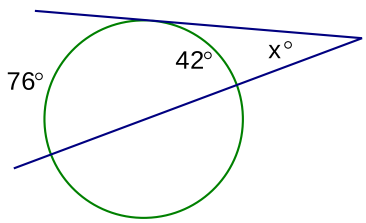
$$118=\frac{1}{2}y$$
$$236^{\circ}=y$$

Ex 2: Find the value of *x*.



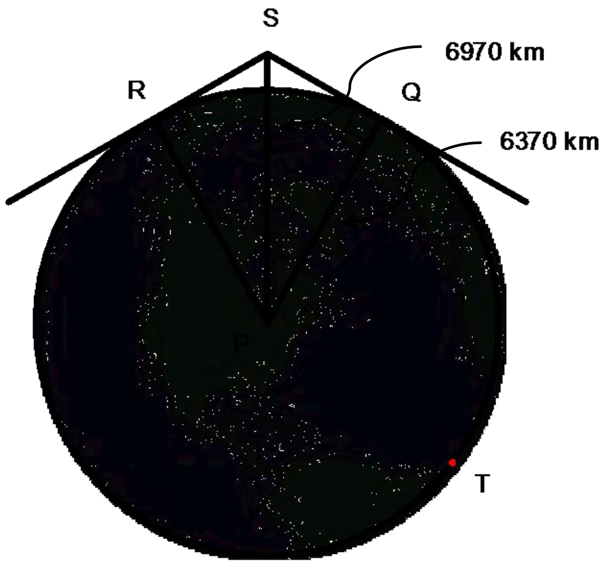
$$x=\frac{1}{2}\left(63+89\right)=76^{\circ}$$

Ex 3: Find the value of *x*.



$$x=\frac{1}{2}\left(76-42\right)=17^{\circ}$$

Ex 4: The Space Shuttle typically orbits at a height of 600 kilometers. Suppose an astronaut takes a picture from point *S*. What is the measure of RQ? (Round your answer to the nearest tenth)



$$m\angle RSP = \sin^{-1}\left(\frac{6370}{6970}\right) = 66.1^\circ$$

$$m\angle RSQ = 2m\angle RSP = 2(66.1^\circ) = 132.2^\circ$$

$$m\angle RSQ = \frac{1}{2}((360 - x) - x)$$

$$132.2^\circ = \frac{1}{2}((360 - x) - x)$$

$$132.2^\circ = \frac{1}{2}(360 - 2x)$$

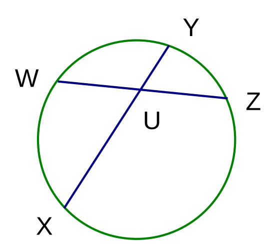
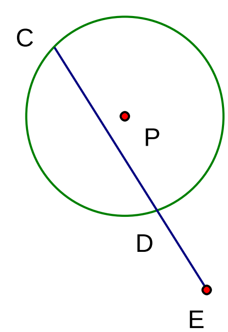
$$132.2^\circ = 180 - x$$

$$x = 47.8^\circ$$

Section:	**10 – 6 Find Segment Lengths in Circles**
Essential Question	What are some properties of chords, secants, and tangents to a circle?

Warm Up:

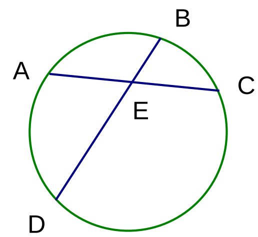
Key Vocab:

Segments of a Chord	When two chords intersect in the interior of a circle, each chord is divided into two segments called *segments of the chord.*	 $\overline{WU}$ and $\overline{UZ}$ are segments of chord $\overline{WZ}$. $\overline{XU}$ and $\overline{UY}$ are segments of chord $\overline{XY}$.
Secant Segment	A segment that contains a chord of a circle and has exactly one endpoint outside the circle.	 $\overline{CE}$ is a *secant segment*. $\overline{DE}$ is the *external segment.*
External Segment	The part of a secant segment that is outside the circle	

Theorems:

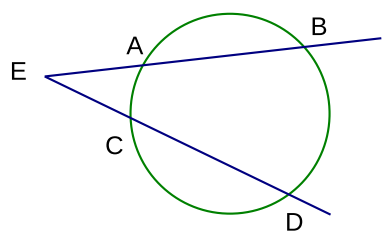
Segments of Chords Theorem

If	Then
two chords intersect in the interior of a circle,	then the product of the lengths of the segments of one chord is equal to the product of the lengths of the segments of the other chord.
	$EA \cdot EC = BE \cdot ED$



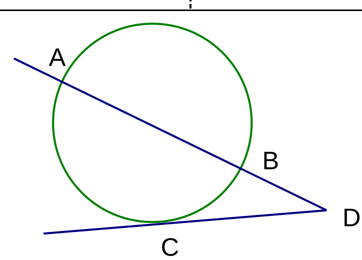
Segment of Secants Theorem

If	Then
two secant segments share the same endpoint outside a circle,	the product of the lengths of one secant segment and its external segment equals the product of the lengths of the other secant segment and its external segment.
	$EA \cdot EB = EC \cdot ED$



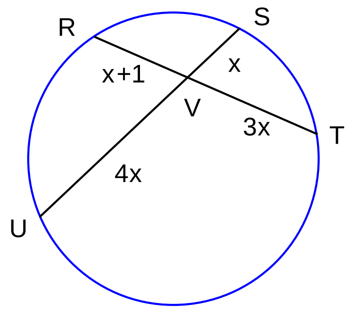
Segments of Secants and Tangents Theorem

If	Then
a secant segment and a tangent segments share an endpoint outside a circle,	the product of the lengths of the secant segment and its external segment equals the square of the length of the tangent segment.
	$CD^2 = DB \cdot AD$



Show:

Ex 1: Find *RT* and *SU*.

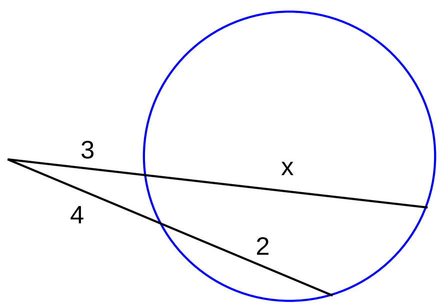


$3x(x+1) = x(4x)$
$3x^2 + 3x = 4x^2$
$0 = x^2 - 3x$
$0 = x(x-3)$
$x = 0, 3$

$RT = [(3)+1]+[3(3)] = 13$

$SU = [4(3)]+[3] = 15$

Ex 2: What is the value of *x?*



$3(3+x) = 4(4+2)$
$9 + 3x = 24$
$3x = 15$
$x = 5$

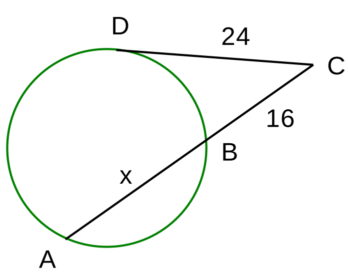
A.) 2

B.) $2\frac{2}{3}$

C.) 5

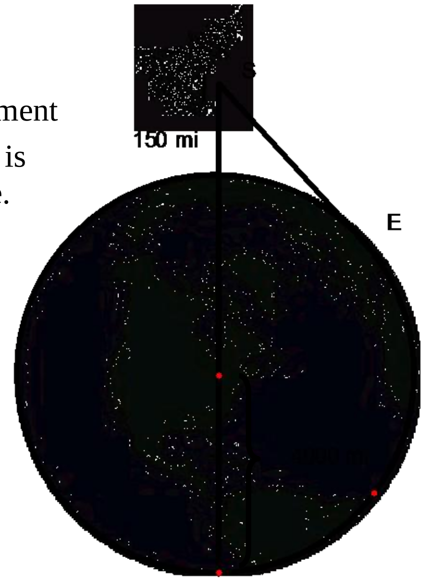
D.) 8

Ex 3: Use the figure to find *AB*.



$24^2 = 16(x+16)$
$576 = 16x + 256$
$320 = 16x$
$20 = x$

Ex 4: Suppose that Space Shuttle is positioned at point *S*, 150 miles above the Earth. What is the length of line segment $\overline{ES}$ from the Earth to the shuttle? (The radius of the Earth is about 4000 miles.) Round your answer to the nearest mile.

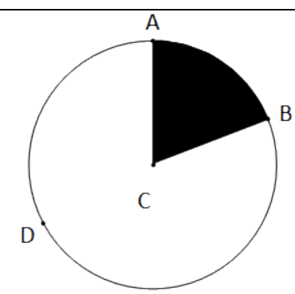


$$ES^2 = 150(150 + 8000)$$
$$ES^2 = 122500$$
$$ES \approx 1106 \text{ miles}$$

Section:	**10 – 7 Write and Graph Equations of Circles**
Essential Question	What do you need to know to write the standard equation of a circle?

Warm Up:

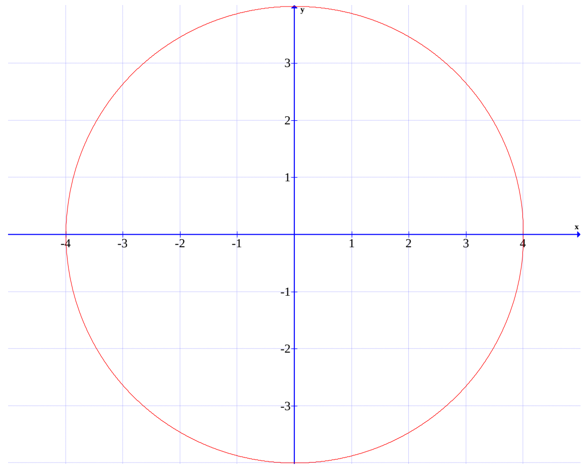
Key Vocab:

Standard Equation of a Circle	The standard equation of a circle with center (h, k) and radius r is $(x-h)^2+(y-k)^2=r^2$	
Area of a Circle	$A=\pi r^2$	
Sector of a Circle	A region of a circle that is bounded by two radii and an arc of the circle.	
Area of a Sector	$A=\frac{mAB}{360}\pi r^2$	

Show:

Ex 1: Write the equation of the circle shown.

$x^2 + y^2 = 16$



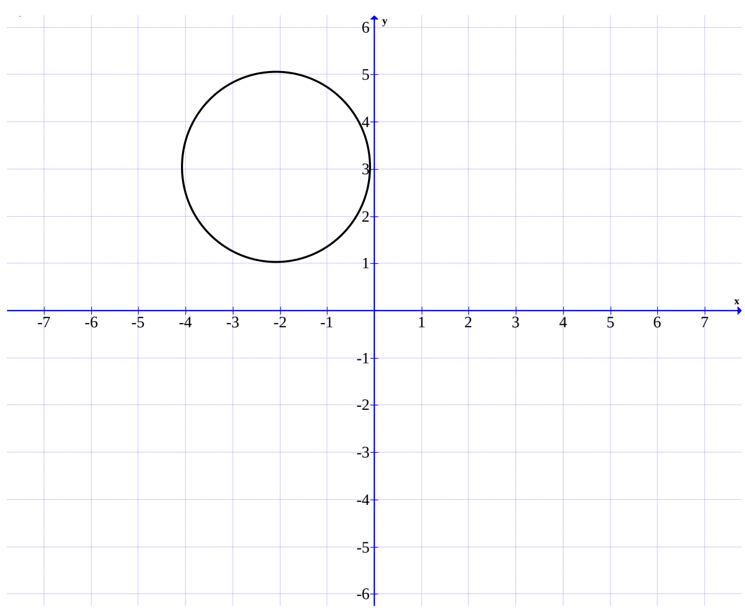
Ex 2: Write the standard equation of a circle with center $(-2,3)$ and radius 3.8.

$(x+2)^2 + (y-3)^2 = 14.44$

Ex 3: The point $(8,-1)$ is on a circle with center $(4,2)$. Write the standard equation of the circle.

$(x-4)^2 + (y-2)^2 = 25$

Ex 4: The equation of a circle is $(x+1)^2 + (y-3)^2 = 4$. Graph the circle.



Ex 5: In $\odot O$ with radius 9, $m\angle AOB = 120$. Find the area of the circle and the areas of each of the sectors.

$A_{circle} = \pi \bullet 9^2$
$A_{circle} = 81\pi \approx 254.5$

$A_{sector1} = \frac{120}{360}\pi \bullet 9^2$
$A_{sector1} = 27\pi \approx 84.8$

$A_{sector2} = \frac{240}{360}\pi \bullet 9^2$
$A_{sector2} = 54\pi \approx 169.6$

Ex 6: Find the area of the region bounded by $\overline{AB}$ and AB.

$A_{sector} = \frac{120}{360}\pi \bullet 6^2 = 12\pi$

− $A_{\Delta ABO} = \frac{1}{2} \bullet 3 \bullet 6\sqrt{3} = 9\sqrt{3}$

$A_{total} = 12\pi - 9\sqrt{3} \approx 22.1$