Ch 1 Points, Lines, Planes and Angles

Undefined Terms

1) Point: a location;
→ no dimensions

2) Line: an infinitely straight path
→ one dimension

3) Plane: an infinitely flat surface
→ two dimensions

Defined Terms

1) Line Segment: part of a line
finite

2) Ray: half of a line
infinite in one direction

3) Opposite Rays: rays that form a line

Symbolism

$\overleftrightarrow{AB}$ line AB

$\overline{AB}$ line segment AB

$\overrightarrow{AB}$ ray AB

AB length of line segment AB

$\angle ABC$ angle ABC

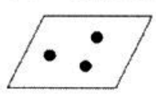
$m\angle ABC$ measure of angle ABC

Collinear Points

Lie in the same line

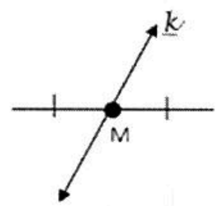
Coplanar Points

Lie in the same plane



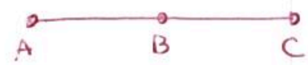
Midpoint/Segment Bisector

Point M is called the midpoint of the segment.



Line k is called the segment bisector

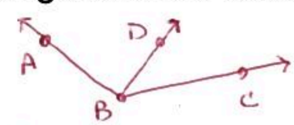
Segment Addition Postulate:



If B is between A and C, then $\boxed{AB + BC = AC}$

→ The sum of the parts equals the whole ←

Angle Addition Postulate:



If D is in the interior of ∠ABC, then

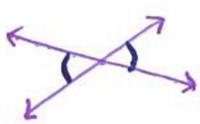
$\boxed{m\angle ABD + m\angle DBC = m\angle ABC}$

→ The sum of the parts equals the whole ←

Vertical Angles:

angles formed by two intersecting lines, they share a vertex but no sides

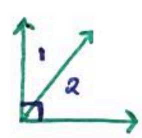
They are always ≅!



Complementary Angles:

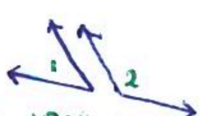
Two angles whose measures have a total of 90°

$m\angle 1 + m\angle 2 = 90°$



Supplementary Angles:

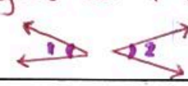
Two angles whose measures have a total of 180°



$m\angle 1 + m\angle 2 = 180°$

Congruent Angles:

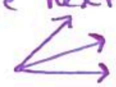
angles with equal measures



$\angle 1 \cong \angle 2$

Adjacent Angles:

angles that are next to each other



Linear Pair:

Adjacent supplementary angles → together, they form a line

Ch 2 Reasoning and Proof

Conditional Statement: "If- then" statements

Converse: the reverse of a conditional → switch the "if" and "then" statements

Inverse: The negation of a conditional

Contrapositive: The negation of a converse

Bi-conditional Statement: an IFF (if and only if) statement formed by combining a true conditional with its true converse

Reflexive Property of eq.

$a = a$

every # equals itself

Symmetric Prop. of eq.

If $a=b$, then $b=a$

Transitive Prop of eq.

If $\underbrace{a=b \text{ and } b=c}_{a=c}$, then $a=c$

Reflexive Property of ≅

$\overline{AB} \cong \overline{AB}$, $\angle ABC \cong \angle XYZ$

Symmetric Prop. of ≅

If $\overline{AB} \cong \overline{BC}$, then $\overline{BC} \cong \overline{AB}$

Transitive Prop of ≅

If $\underbrace{\overline{AB} \cong \overline{BC} \text{ and } \overline{BC} \cong \overline{CD}}$, then $\overline{AB} \cong \overline{CD}$

Point, Line and Plane Postulates:

1) Through any two points there is exactly one line.
2) A line contains at least 2 points
3) The intersection of 2 lines is exactly one point
4) Through any 3 NONCOLLINEAR points there is exactly one plane
5) A plane contains at least 3 NONCOLLINEAR points
6) If 2 points lie in a plane, then the line containing them does too
7) The intersection of 2 planes is a line (infinite intersection)

Right Angles ≅ Thm

All Right Angles are ≅

Vertical Angles ≅ Thm

All vertical Angles are ≅

Congruent Supplements Thm.

Two angles supplementary to the same angle are congruent

-OR-

Two angles supplementary to congruent angles are congruent

Cong. Complements Thm.

Two angles complementary to the same angle are congruent

-OR-

Two angles complementary to congruent angles are congruent

Linear Pair Postulate

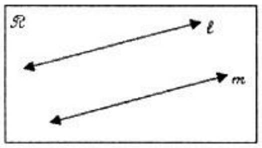
If Two angles form a linear pair, then they are supplementary

Ch 3 Parallel and Perpendicular Lines

Parallel Lines

Symbol ∥

Two lines that do not intersect and are coplanar



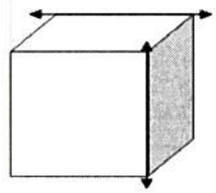
Perpendicular Lines

Symbol ⊥

Two lines that intersect to form four right angles

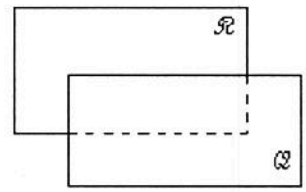
Skew Lines

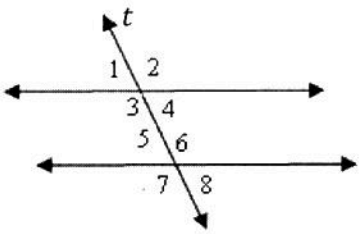
Lines that do not intersect and are NOT coplanar



Parallel Planes

Two planes that do not intersect





Line t is called the transversal

∠1 and ∠5 are Corresponding angles.

∠3 and ∠6 are alternate interior angles.

∠2 and ∠7 are alternate exterior angles.

∠4 and ∠6 are Consecutive interior angles.

If two **parallel** lines are cut by a transversal:

1) Alternate interior, alternate exterior and Corresponding angles are congruent.
2) Consecutive interior angles are supplementary.

Methods to prove that two lines are parallel:

1) Prove alt. interior angles are ≅
2) Prove alt. exterior angles are ≅.
3) Prove corresponding angles are ≅.
4) Prove Cons. Interior angles are supplementary

Slope:

$m = \frac{rise}{run} = \frac{y - y_1}{x - x_1}$

"rate of change"

Horizontal Slope:

$m = 0$

Vertical Slope:

$m = \emptyset$ = No Slope

Slope-Intercept form:

$y = mx + b$

Standard form:

$Ax + By = C$

Point- Slope form:

$y - y_1 = m(x - x_1)$

Vertical Line: $x = a$

Horizontal Line: $y = b$

Writing an equation parallel to given line:

parallel lines have EQUAL SLOPES

Writing an equation perpendicular to given line:

perpendicular lines have OPPOSITE RECIPROCAL SLOPES

Ch 4 Congruent Triangles

Types of Triangles by Sides

Scalene: NO ≅ sides

Isosceles: at least TWO ≅ sides

Equilateral: All ≅ sides (Also equiangular)

Types of Triangles by Angles

Acute: All ∠ measures (<) 90°

Right: ONE ∠ measure (=) 90°

Obtuse: ONE ∠ measure (>) 90°

Equiangular: All ∠ measures (=) 60°

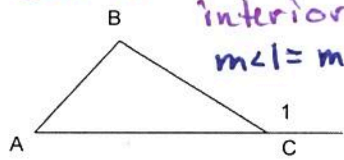
Δ Sum Thm.

every Δ has a total of 180°

Ext Angle Thm.

The measure of an ext. ∠ is = to the sum of the remote interior two.

$m\angle 1 = m\angle A + m\angle B$



Components of an Isosceles Triangle

legs: The congruent sides.

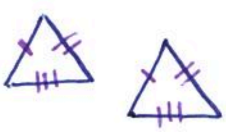
vertex angle: The angle formed by the legs.

base: The third side (not a leg)

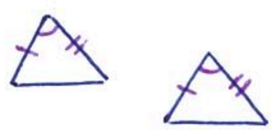
base angles: The two angles adjacent to the base.

Methods to prove that two triangles are congruent (Draw a picture for each)

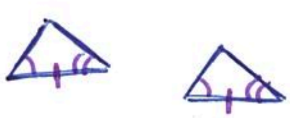
1) SSS ≅



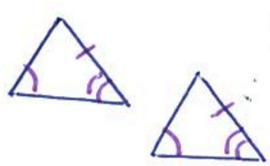
2) SAS ≅



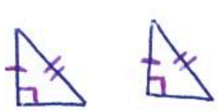
3) ASA ≅



4) AAS ≅



5) HL ≅ *only for RT Δ's



IS FOLLOWED BY

CPCTC means Corresponding Parts of Congruent Δ's are Congruent

Base Angles Theorem

If a Δ is isosceles, then its base angles are ≅

Base Angles Theorem Converse:

If two angles are ≅, then the sides opposite them are ≅

If a triangle is equilateral, then it is equiangular.

If a triangle is equiangular, then it is equilateral.

Each angle in an equiangular triangle measures 60°.

Ch 5 Relationships within Triangles

Mid-segment theorem

The segments connecting the midpoints of two sides of a Δ are:

a) ∥ to the 3rd side

b) half the length of the 3rd side

Perpendicular Bisector

A segment, ray, line or plane that is perpendicular to a segment at its midpoint.

Altitude

The perpendicular segment from one vertex of the triangle to the line that contains the opposite side.

Median

A segment from one vertex of the triangle to the midpoint of the opposite side

Points of Concurrency

The circumcenter is where the perpendicular bisectors of a triangle meet.

The incenter is where the angle bisectors meet.

The centroid is where the medians meet.

The orthocenter is where the altitudes meet.

Triangle Inequality Theorem

Given three lengths, a triangle is formed when the sum of any two lengths is greater than the third length.

SM + MED > LARGE

In any triangle…

The longest side is across from the largest ∠.

The shortest side is across from the smallest ∠.

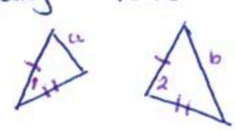
Circumcenter Theorem: The perpendicular bisectors of a triangle meet at a point equidistant from the vertices of a triangle.

Incenter Theorem: The angle bisectors of a triangle meet at a pt that is equidist. from the sides of a triangle.

Centroid Theorem: The medians of a triangle meet at a point 2/3 the distance from each vertex to the midpoint of the opposite side.

Hinge Theorem: "SAS Inequality"

If two pair of ≅ sides AND ONE pair of unequal angles, then the sides opposite the angles have the same inequality



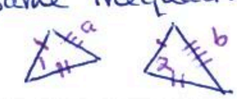
if $m\angle 1 < m\angle 2$
then $a < b$

Hinge Theorem Converse:

"SSS Inequality"

If two pair of ≅ sides AND ONE pair of unequal sides, then the angles opposite the unequal sides have the same inequality.

if $a < b$
then $m\angle 1 < m\angle 2$



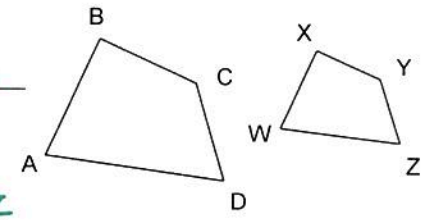
Ch 6 Similarity

Similar Polygons

Two polygons such that the corresponding angles are Congruent

and the corresponding sides are proportional.

Symbol for Similarity: ~ Similarity Statement: ABCD ~ WXYZ



Scale Factor

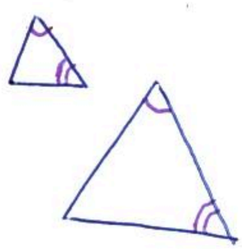
Ratio of corresponding side lengths.

Perimeter of Similar Polygons Theorem

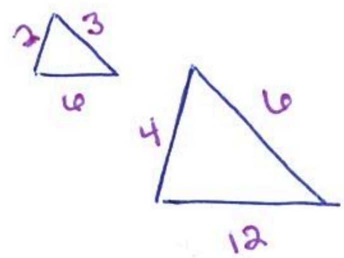
If two polygons are similar, then the ratio of their perimeters is equal to the ratios of corresponding side lengths.

Methods to prove that two triangles are congruent (Draw a picture/example for each)

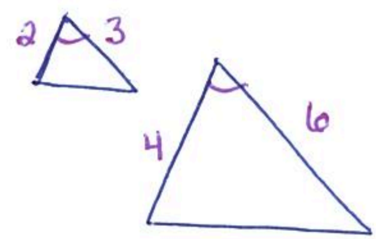
1) AA ~



2) SSS ~



3) SAS ~



Ch 7 Right Triangles & Trigonometry

Pythagorean Theorem

$a^2 + b^2 = c^2$

where c is the hypotenuse, and the legs are a and b.

Converse of the Pythagorean Theorem

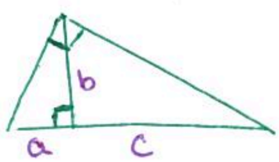
An acute triangle is formed when $c^2 < a^2 + b^2$

A right triangle is formed when $c^2 = a^2 + b^2$

An obtuse triangle is formed when $c^2 > a^2 + b^2$

Geometric Mean Altitude Theorem

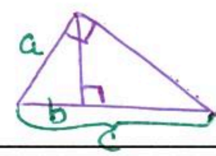
The altitude is the geometric mean of the two segments of the hypotenuse.



$\frac{a}{b} = \frac{b}{c}$

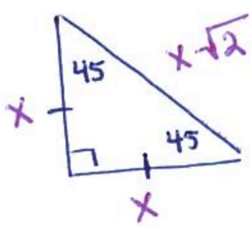
Geometric Mean Leg Theorem

The leg is the geometric mean of the hypotenuse and the segment of the hypotenuse adjacent to the leg.



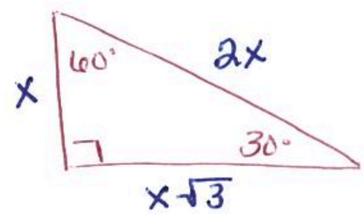
$\frac{a}{b} = \frac{c}{a}$

45°-45°-90° Triangle Theorem



Isosceles Right Δ

30°-60°-90° Triangle Theorem



Trig Ratios

Used to find sides.

Sin θ= $\frac{opp.}{hyp.}$

Tan θ= $\frac{opp.}{adj.}$

Cos θ= $\frac{adj.}{hyp.}$

$S = \frac{O}{H}$ $C = \frac{A}{H}$ $T = \frac{O}{A}$

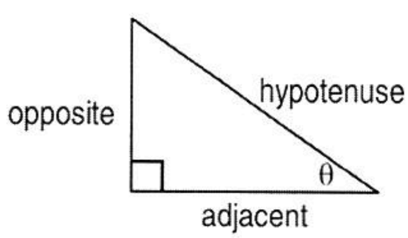
Inverse Trig Ratios

Used to find angles.

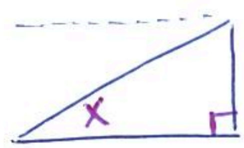
$\theta = \sin^{-1}(opp/hyp)$

$\theta = \cos^{-1}(adj/hyp)$

$\theta = \tan^{-1}(opp/adj)$

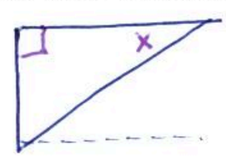


Angle of Elevation



• Alt. Int. Angles
• Always ≅

Angle of Depression



To solve a right triangle you must find: ALL Sides **and ALL** angles

Ch 8 Polygons & Quadrilaterals

Polygon Interior Angle Sum Theorem

$(n-2)180$

The Polygon EXTERIOR Angle Sum always equals $360°$.

Regular Polygon

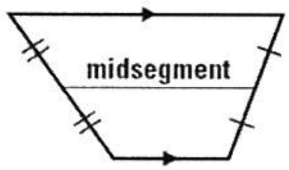
A polygon with all sides congruent and all angles congruent.

Example: Stop Sign

Square

$A = \frac{1}{2}aP$

Trap. Midseg. Thm

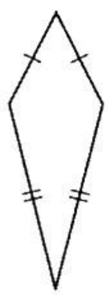


To find the midsegment add the ______ and then divide by ___.

______ →**NO OPPOSITE SIDES PARALLEL**

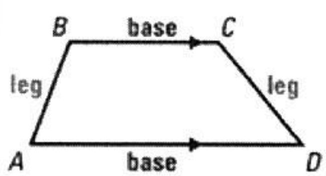
2 pairs of consecutive sides are ≅

One pair of opp. Angles ≅



ONE SET OF OPPOSITE SIDES PARALLEL

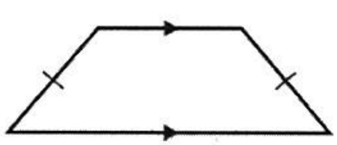
1) $A = \frac{1}{2}h(b_1+b_2)$



2) ______________________

Pair of ≅ legs and 2 pairs of ≅ base angles.

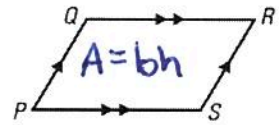
Diagonals are congruent.



isosceles trapezoid

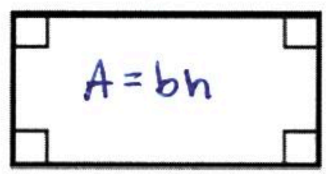
BOTH SETS OF OPPOSITE SIDES ARE PARALLEL

Parallelogram



1. Opposite sides are parallel
2. Opposite sides are congruent
3. Opposite angles are congruent.
4. Consecutive angles are supplementary
5. Diagonals bisect each other

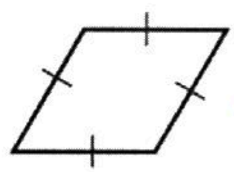
Rectangle



All properties of a parallelogram plus

1. All angles are ______________
2. Diagonals are ______________

Rhombus



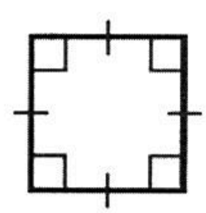
$A = \frac{1}{2}d_1d_2$

All properties of a parallelogram plus

1. All sides are ______________
2. Diagonals are ______________
3. Diagonals bisect the ______________

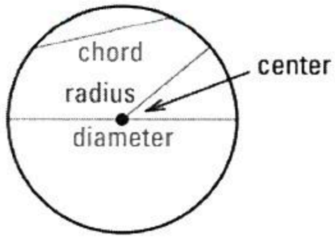
Square

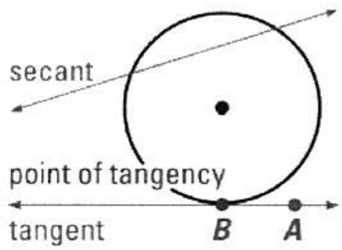
All properties of parallelogram, rhombus, and rectangle.



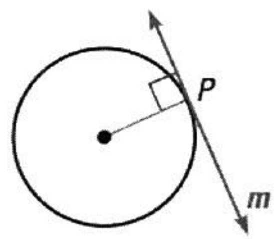
Ch 10 Circles

Parts of a Circle



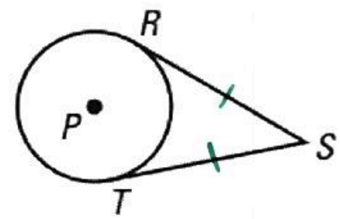


A line is tangent to a circle IFF the line is perpendicular to a radius of a circle at its intersection on the circle.



Line m is tangent to $\odot Q$ if and only if $m \perp \overline{QP}$.

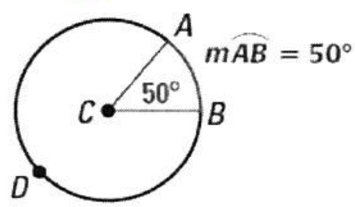
Tangent segments from a common external point are congruent.



If $\overline{SR}$ and $\overline{ST}$ are tangent segments, then $\overline{SR} \cong \overline{ST}$.

The measure of a minor arc is equal to the measure of the central angle.

The measure of a major arc is the difference between 360° and the measure of the related minor arc.

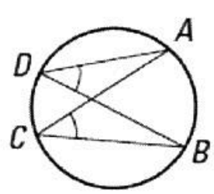


$m\widehat{ADB} = 360° - 50° = 310°$

The measure of the entire circle is 360°.

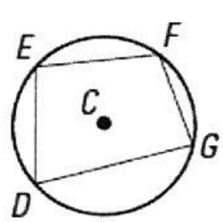
The measure of a semicircle is 180°.

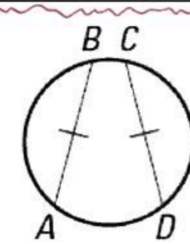
If two inscribed angles of a circle intercept the ______ arc, then the angles are __________.



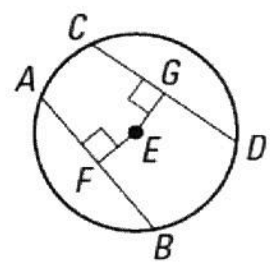
$\angle ADB \cong \angle ACB$

A quadrilateral can be inscribed in a circle IFF its ____________ angles are ______________.

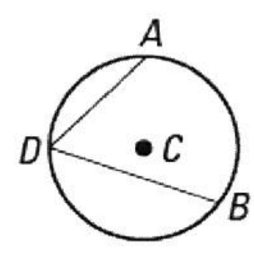




$\widehat{AB} \cong \widehat{CD}$ **if and only if** $\overline{AB} \cong \overline{CD}$.



$\overline{AB} \cong \overline{CD}$ **if and only if** $EF = EG$.



$m\angle ADB = \frac{1}{2} m\widehat{AB}$