Name: ________________________

6.1 Functions

Essential Question:

- **What is a function?**

Relation	A set of ordered pairs such that each member of the domain corresponds to *__at least__* one member of the range
Function	A set of ordered pairs such that each member of the domain corresponds to *__exactly__* one member of the range
Domain	The set of *x*'s (inputs, abscissas, independents) that when plugged into the function give REAL NUMBER results
Range	The set of *y*'s (outputs, ordinates, dependents)
Ordered Pairs	(x, y) (input, output) (Independent, Dependent)

Examples:

Indicate whether the set defines a function. If it does, state the domain and range of the function.

1. {(6, 8), (7, 9), (8, 10), (9, 11)}

A function
Domain = {6, 7, 8, 9}
Range = {8, 9, 10, 11}

2. {(1, 3), (1, 4), (1, 5), (1, 6)}

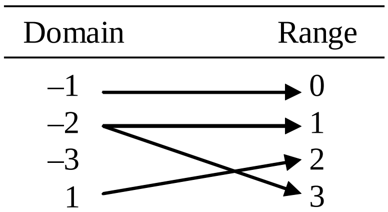
Not a function

3. {(1, 6), (2, 6), (3, 6), (4, 6)}

A function
Domain = {1, 2, 3, 4}
Range = {6}

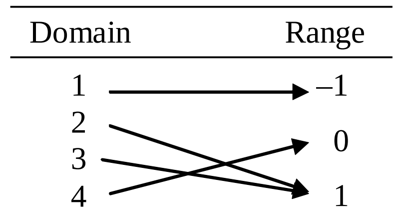
Determine whether each of the following relations is a function. If it is a function, identify the domain and range.

4.



A) Not a Function

5.



A) Function

B) D = {1, 2, 3, 4}

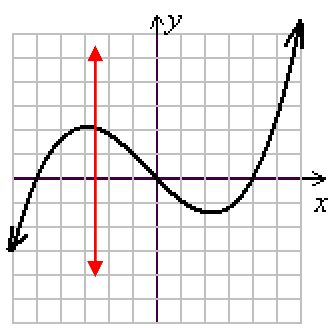
R = {-1, 0, 1}

The Vertical Line Test	If a vertical line intersects a graph in <u>more than one point</u>, the graph is <u>*not*</u> that of a function

Examples:

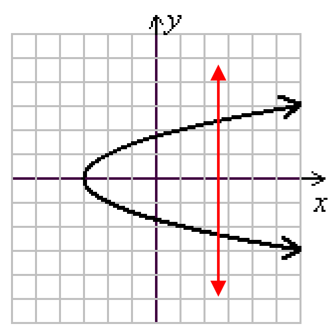
Indicate whether the graph is the graph of a function.

6.



Function – the vertical line only touches the graph in one point

7.



Not a function – the vertical line touches the graph in <u>two</u> places.

Algebraic Function Test	Solve the equation for *y*, then determine if for each *x*-value there is only one corresponding y value. →odd exponents on the *y*-value are functions →even exponents on the *y*-value are NOT functions

Examples:

Indicate whether the equation defines a function with independent variable *x*.

8. $4x^2 + y = -9$

$y = -9 - 4x^2$

function

9. $6x - y^2 = 7$

$y^2 = 6x - 7$

$y = \pm\sqrt{6x-7}$

Not a function – there are TWO *y*-values for the same *x*-value

10. $-3x + y^3 = 8$

$y = \sqrt[3]{3x+8}$

Function

11. $-2x + y^2 = 3$

$y^2 = 2x + 3$

$y = \pm\sqrt{2x+3}$

Not a function

Function Notation	$f(x)$ **replaces the *y* variable** **read "*f* of *x*"** **Example:** $y = x^3 + 1$ **is the same as** $f(x) = x^3 + 1$

Examples:

Evaluate each of the following functions.

12. Find the value of $f(2)$ if $f(x) = -x + 8$.

$f(2) = -2 + 8$

$f(2) = 6$

13. Find $f(a-2)$ if $f(x) = -x - 5$.

$f(a-2) = -(a-2) - 5$

$f(a-2) = -a + 2 - 5$

$f(a-2) = -a - 3$

Finding the Domain of a Function	
Fractions	Rational means fractional → denominator of a fraction $\neq 0$
Radicals	Expressions under square roots must be positive or zero →You cannot substitute any value for a variable in the radical that would cause the radicand to be negative. →Negatives in a square root result in imaginary numbers.
• Any other functions will *not* have restrictions on the domain unless otherwise noted.	

Examples:

Find the domain of the following functions. Write answers in interval notation.

14. $g(x)=\sqrt{x+4}$

$$x+4\geq 0$$
$$x\geq -4$$

$$[-4,\infty)$$

15. $k(x)=\dfrac{2x-13}{x^2-2x-35}$

$$x^2-2x-35\neq 0$$
$$(x+5)(x-7)\neq 0$$
$$x\neq -5,7$$

$$(-\infty,-5)\cup(-5,7)\cup(7,\infty)$$

16. $g(x)=\sqrt{2x-3}$

$$2x-3\geq 0$$
$$x\geq \frac{3}{2}$$

$$\left[\frac{3}{2},\infty\right)$$

17. $k(x)=\dfrac{x+2}{x^2-16}$

$$x^2-16\neq 0$$
$$(x+4)(x-4)\neq 0$$
$$x\neq -4,4$$

$$(-\infty,-4)\cup(-4,4)\cup(4,\infty)$$

Difference Quotient	$\frac{f(x+h)-f(x)}{h}$

Example:

Evaluate the difference quotient for each of the following.

18. $f(x)=2x+10$

$$\begin{aligned}\frac{2(x+h)+10-(2x+10)}{h} &= \frac{2x+2h+10-2x-10}{h}\\ &= \frac{2h}{h}\\ &= 2\end{aligned}$$

19. $f(x)=x^2-9$

$$\begin{aligned}\frac{(x+h)^2-9-(x^2-9)}{h} &= \frac{x^2+2xh+h^2-9-x^2+9}{h}\\ &= \frac{2xh+h^2}{h}\\ &= \frac{h(2x+h)}{h}\\ &= 2x+h\end{aligned}$$

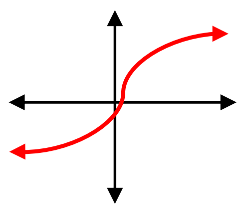
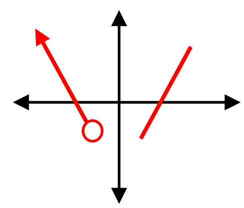
20. $f(x)=x^2+7$

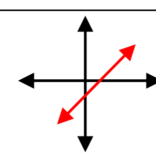
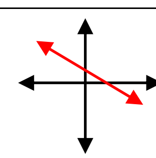
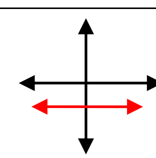
$$\begin{aligned}\frac{(x+h)^2+7-(x^2+7)}{h} &= \frac{x^2+2xh+h^2+7-x^2-7}{h}\\ &= \frac{2xh+h^2}{h}\\ &= \frac{h(2x+h)}{h}\\ &= 2x+h\end{aligned}$$

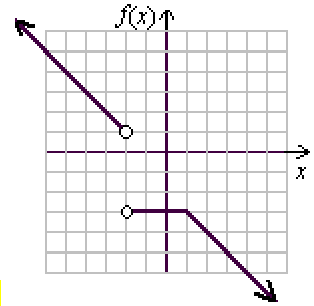
6.2 Graphing Functions

Essential Question(s):

- **How do you graph piecewise-defined functions?**

x – intercept	zero, root, solution Where graph crosses the x – axis Where the *y*-value is zero : (*x*, 0)
y – intercept	Where graph crosses the y – axis Where the *x*-value is zero : (0, *y*)
Continuous Function	A function that has no breaks, gaps, or holes. Example:  Non-Example: 
Discontinuity	Break, gap, or hole in the graph Any place where the function is not continuous.

Characteristics of a function	
On an open interval, for any *a* and *b* in the interval:	
1. If $a < b$ while $f(a) < f(b)$ for all values, then the function is **increasing** Translation: as *x* increases, *y* increases	
2. If $a < b$ while $f(a) > f(b)$ for all values, then the function is **decreasing** Translation: as *x* increases, *y* decreases	
3. If for all *a* and *b* in that interval $f(a) = f(b)$, then the function is **constant** Translation: as *x* increases, *y*-value does NOT change	



Examples:

Use the following graph to answer questions 1-8:

1. Find the domain of *f*.

$(-\infty,-2)\cup(-2,\infty)$

2. Find the range of *f*.

$(-\infty,-3)\cup(1,\infty)$

3. Find the *x*-intercept(s).

x intercept: none

4. Find the *y*-intercept(s).

y intercept: -3

5. Find the intervals over which *f* is increasing.

none

6. Find the intervals over which *f* is decreasing.

$(-\infty,-2)\cup(1,\infty)$

7. Find the intervals over which *f* is constant.

$(-2,1)$

8. Find any points of discontinuity.

$x=-2$

9. $f(x)=\dfrac{3x+12}{2x-8}$

a. Find the domain of *f*.

$$2x-8\neq 0$$
$$2x\neq 8$$
$$x\neq 4$$

b. Find the *x*-intercept.

$$0=\frac{3x+12}{2x-8}$$
$$0=3x+12$$
$$-3x=12$$
$$x=-4$$

c. Find the *y*-intercept.

$$f(0)=\frac{3(0)+12}{2(0)-8}$$
$$f(0)=\frac{12}{-8}=-1.5$$

10. $f(x)=\dfrac{x^2-1}{x^2+100}$

a. Find the domain of *f*.

$$x^2+100=0$$
$$\{\text{all real numbers}\}$$

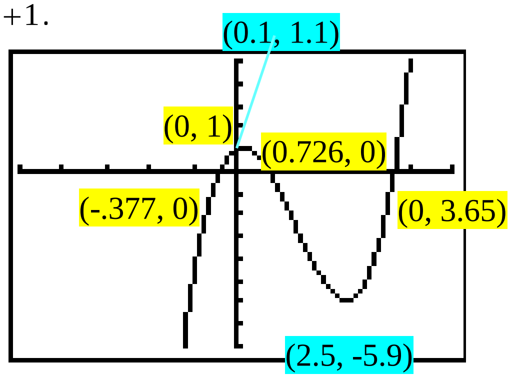
b. Find the *x*-intercept.

$$0=\frac{x^2-1}{x^2+100}$$
$$0=x^2-1$$
$$1=x^2$$
$$\pm 1=x$$

c. Find the *y*-intercept.

$$f(0)=\frac{0^2-1}{0^2+100}$$
$$f(0)=\frac{-1}{100}=-.01$$

11. **a.** Use the graphing calculator sketch the graph of $y = x^3 - 4x^2 + x + 1$.



b. Identify all *x*- and *y*-intercepts.

Use 2nd →Trace →2: Zero to find the *x*-intercepts.

Use 2nd →Trace →1: Value to find the *y*-intercepts.

c. Identify all maximum and minimum values.

Use 2nd →Trace →3: Minimum→Identify: Left Bound, Right Bound, and Guess values

Use 2nd →Trace →4: Maximum→Identify: Left Bound, Right Bound, and Guess values

d. Identify the intervals over which the graph is increasing and decreasing.

Increasing: $(-\infty, 0.1) \cup (2.5, \infty)$

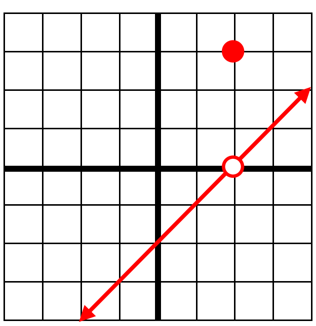
Decreasing: $(0.1, 2.5)$

Piecewise function	A function that is defined by two (or more) equations over a specified domain

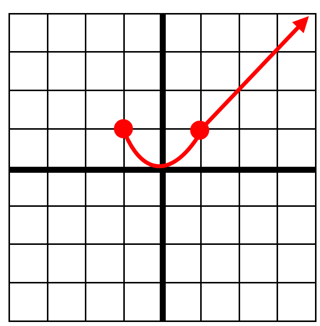
Examples:

Make a sketch of the following functions

12. $f(x) = \begin{cases} x - 2, x \neq 2 \\ 3, x = 2 \end{cases}$



13. $f(x) = \begin{cases} x^2, -1 \leq x \leq 1 \\ x, x > 1 \end{cases}$



14. Given: $f(x)=\begin{cases} x+2 & \text{if } -4\le x<1 \\ -x & \text{if } 1\le x\le 5 \end{cases}$

a. Find *f*(–4), *f*(1), and *f*(5).

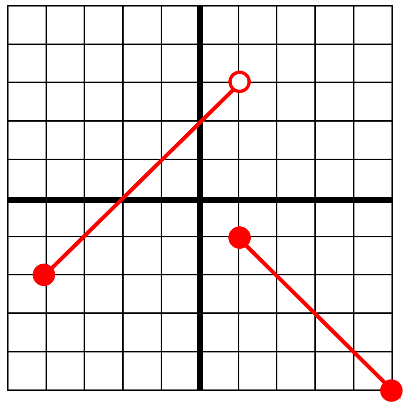
$f(-4)=-4+2$

$f(-4)=-2$

$f(1)=-1$

$f(5)=-5$

b. Sketch the graph of *f*.



c. Find the domain, range, and all points of discontinuity.

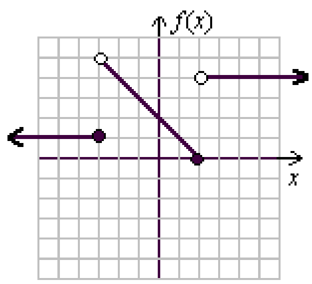
Domain: $[-4,5]$

Range: $[-5,3)$

Discontinuous at x = 1

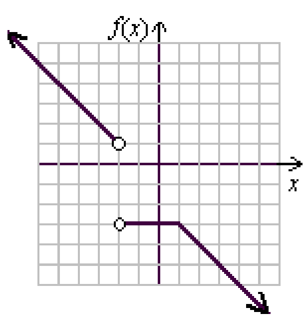
15. Find a piecewise definition for the graph of *f*(x).

$$f(x)=\begin{cases} 1 & x\le -3 \\ -x+2 & -3<x\le 2 \\ 4 & x>2 \end{cases}$$



16. Find a piecewise definition for the graph of *f*(x).

$$f(x)=\begin{cases} -x-1 & x<-2 \\ -3 & -2<x\le 1 \\ -x-2 & x>1 \end{cases}$$



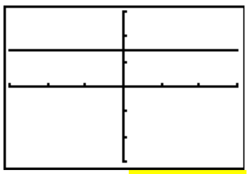
6.3 Transformations of Functions

Essential Question(s):

- **How do you graph vertical and horizontal shifts, reflections, and size transformations of functions?**
- **What are even and odd functions?**

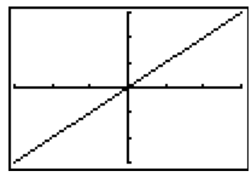
Sketch graphs for each of the following **parent functions**. Find their domains and ranges, and determine if they are even, odd, or neither.

Constant Function: $f(x) = c$



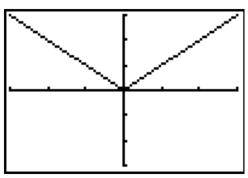
Domain: $(-\infty, \infty)$
Range: c
Even function

Identity Function: $f(x) = x$



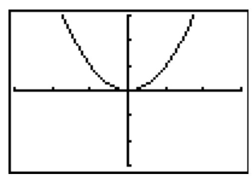
Domain: $(-\infty, \infty)$
Range: $(-\infty, \infty)$
Odd function

Absolute Value Function: $f(x) = |x|$



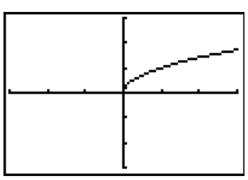
Domain: $(-\infty, \infty)$
Range: $[0, \infty)$
Even function

Standard Quadratic Function: $f(x) = x^2$



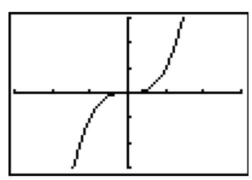
Domain: $(-\infty, \infty)$
Range: $[0, \infty)$
Even function

Square Root Function: $f(x) = \sqrt{x}$



Domain: $(-\infty, \infty)$
Range: $[0, \infty)$
Neither

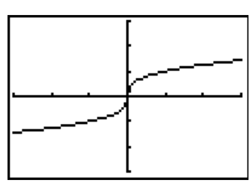
Standard Cubic Function: $f(x) = x^3$



Domain: $(-\infty, \infty)$
Range: $(-\infty, \infty)$
Odd function

Cube Root Function: $f(x) = \sqrt[3]{x}$

Domain: $(-\infty, \infty)$
Range: $(-\infty, \infty)$
Odd function

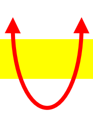


Transformations on Functions	
Let f be a function and c be a positive real number	
Vertical Shifts	• $y=f(x)+c$ is the graph of $y=f(x)$ shifted c units **upward** • $y=f(x)-c$ is the graph of $y=f(x)$ shifted c units **downward**
Horizontal Shifts	• $y=f(x+c)$ is the graph of $y=f(x)$ shifted c units to the **left** • $y=f(x-c)$ is the graph of $y=f(x)$ shifted c units to the **right**
Reflections	• $y=-f(x)$ is the graph of $y=f(x)$ reflected about the **x-axis** • $y=f(-x)$ is the graph of $y=f(x)$ reflected about the **y-axis**
Vertical Stretching and Shrinking	• If $c>1$, $y=cf(x)$ is the graph of $y=f(x)$ **stretched** vertically • If $0<c<1$, $y=cf(x)$ is the graph of $y=f(x)$ **shrunk** vertically →In both cases, **multiply** each **y-coordinate** by c
Horizontal Stretching and Shrinking	• If $c>1$, $y=f(cx)$ is the graph of $y=f(x)$ **shrunk** horizontally • If $0<c<1$, $y=f(cx)$ is the graph of $y=f(x)$ **stretched** horizontally →In both cases, **divide** each **x-coordinate** by c

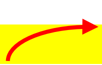
Examples:

Identify the parent function, then determine the type(s) of transformation for each equation.

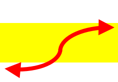
1. $f(x)=x^2+2$

Quadratic Function:  Vertical Shift: Up two places

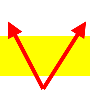
2. $g(x)=\sqrt{x-5}$

Square root Function:  Horizontal Shift: Right five places

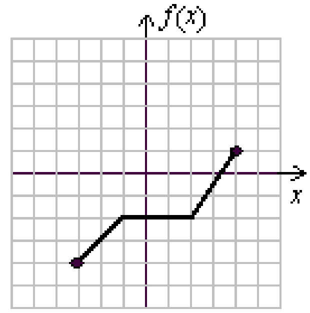
3. $h(x)=\sqrt[3]{x}-4$

Cube Root Function:  Vertical Shift: Down four places

4. $j(x)=-|x+2|$

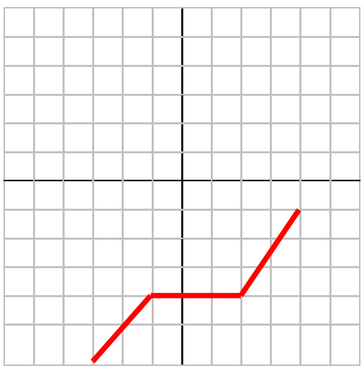
Absolute Value Function:  Horizontal Shift: left two places **AND** *x*-axis Reflection

Use the following to answer questions 1-6:



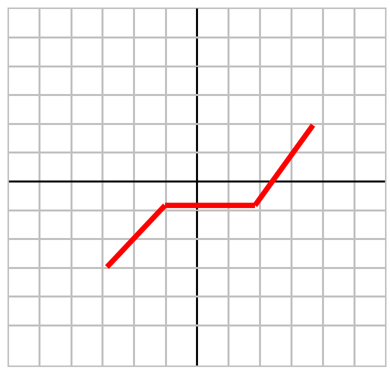
Domain: [–3, 4]
Range: [–4, 1]

5. Graph $h(x) = f(x) - 2$ and state the domain and range of *h*.



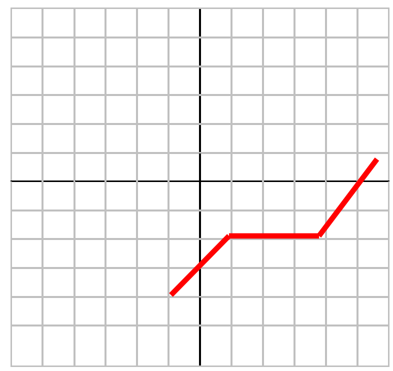
Domain: [–3, 4]
Range: [–6, -1]

6. Graph $h(x) = f(x) + 1$ and state the domain and range of *h*.



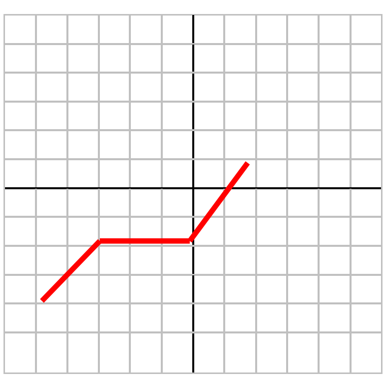
Domain: [–3, 4]
Range: [–3, 2]

7. Graph $h(x) = f(x - 2)$ and state the domain and range of *h*.



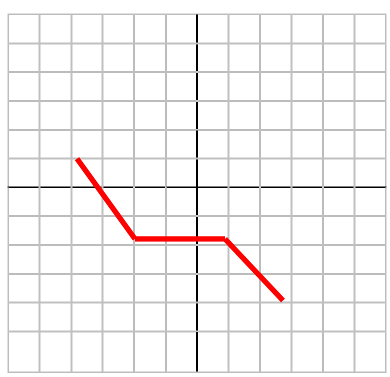
Domain: [–1, 6]
Range: [–4, 1]

8. Graph $h(x) = f(x + 2)$ and state the domain and range of *h*.



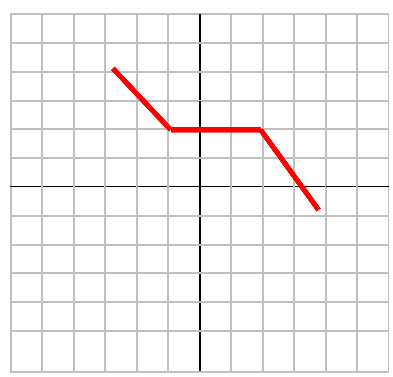
Domain: [–5, 2]
Range: [–4, 1]

9. Graph $h(x) = -f(x)$ and state the domain and range of *h*.



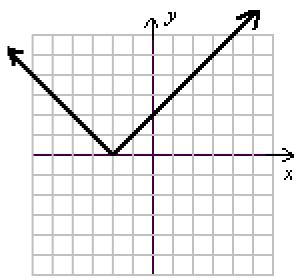
Domain: [–4, 3]
Range: [–4, 1]

10. Graph $h(x) = f(-x)$ and state the domain and range of *h*.



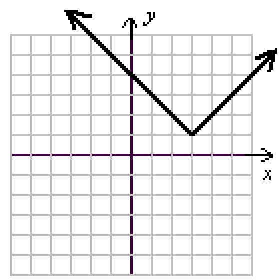
Domain: [–3, 4]
Range: [–1, 4]

11. Determine the function represented by the graph.



$f(x)=|x+2|$

12. Determine the function represented by the graph.



$f(x)=|x-3|+1$

Even Function	• $f(-x)=f(x)$ **for all *x* in the domain of *f*** • **symmetric to the y-axis**
Odd Function	• $f(-x)=-f(x)$ **for all *x* in the domain of *f*** • **Symmetric to the origin**

Examples:

Determine whether the following functions are even, odd or neither.

13. $f(x)=x^2+4$

$$f(-x)=(-x)^2+4$$
$$=x^2+4$$
$$=f(x)$$

Function is Even

14. $f(x)=5x^3-x$

$$f(-x)=5(-x)^3-(-x)$$
$$=-5x^3+1$$
$$=-f(x)$$

Function is Odd

15. $f(x)=x^3+1$

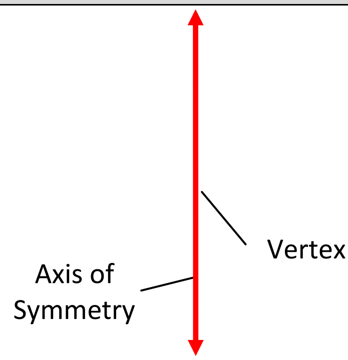
$$f(-x)=(-x)^3+1$$
$$=-x^3+1$$

Function is neither even nor odd

6.4 Quadratic Functions

Essential Question(s):

- **How do you graph quadratic functions?**
- **How do you solve quadratic inequalities?**

<table>
<tr><th colspan="2">Quadratic Equations</th></tr>
<tr><td colspan="2">
</td></tr>
<tr><td rowspan="2">Standard Form</td><td>$f(x)=ax^2+bx+c$</td></tr>
<tr><td>vertex → coordinate: (x, f(x)), where $x=\frac{-b}{2a}$

axis of symmetry → vertical line: $x=\frac{-b}{2a}$

Maximum vs Minimum → Always occurs at the vertex
if a > 0 (positive)→ minimum → graph opens upward
if a < 0 (negative)→ maximum → graph opens downward</td></tr>
<tr><td rowspan="2">Vertex Form</td><td>$f(x)=a(x-h)^2+k$</td></tr>
<tr><td>vertex → coordinate: $(h,\ k)$

axis of symmetry → vertical line: $x=h$

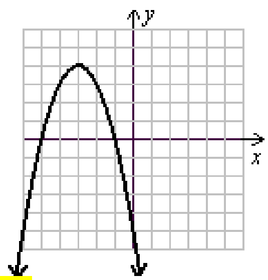
Maximum vs Minimum → Always occurs at the vertex
if a > 0 (positive)→ minimum → graph opens upward
if a < 0 (negative)→maximum → graph opens downward</td></tr>
</table>

Examples:

1. Find the vertex form of the quadratic function $f(x) = x^2 - 20x + 9$.

$$f(x) = (x^2 - 20x + 100) + 9 - 100$$
$$f(x) = (x - 10)^2 - 91$$

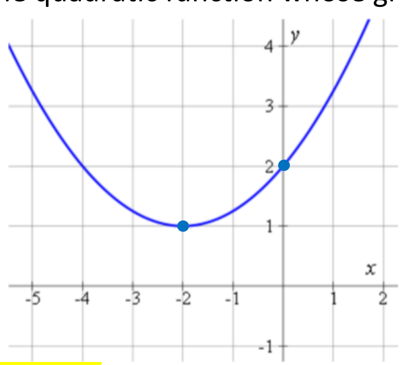
2. Find the standard form equation of the quadratic function whose graph is shown.



$$y = a(x-h)^2 + k$$
$$0 = a(-1+3)^2 + 4$$
$$-4 = 4a$$
$$a = -1$$

$$f(x) = -(x+3)^2 + 4$$
$$f(x) = -(x^2 + 6x + 9) + 4$$
$$f(x) = -x^2 - 6x - 5$$

3. Find the standard form equation of the quadratic function whose graph is shown.



$$y = a(x-h)^2 + k$$
$$2 = a(0+2)^2 + 1$$
$$1 = 4a$$
$$a = \frac{1}{4}$$

$$f(x) = \frac{1}{4}(x+2)^2 + 1$$
$$f(x) = \frac{1}{4}(x^2 + 4x + 4) + 1$$
$$f(x) = \frac{1}{4}x^2 + x + 2$$

4. Given: $g(x) = -3(x+2)^2 + 3$

a. Find the coordinates of the vertex. Is it a maximum or a minimum?

(-2, 3) maximum

b. Find the equation of the axis of symmetry.

$x = -2$

c. Find the domain and range.

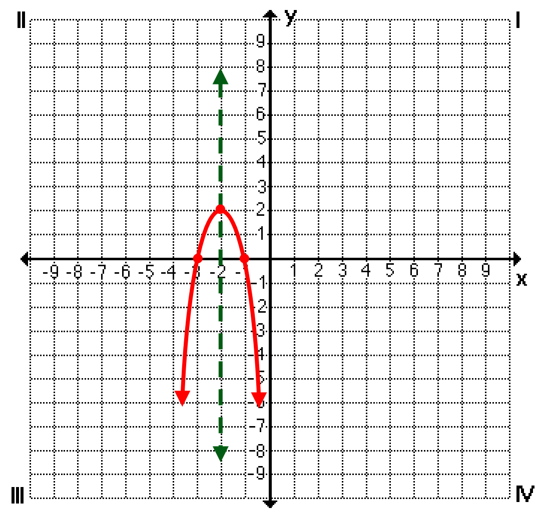
Domain: $(-\infty, \infty)$
Range: $(-\infty, 3]$

d. Find the *x*- and *y*-intercepts.

$0 = -3(x+2)^2 + 3$
$3(x+2)^2 = 3$
$(x+2)^2 = 1$
$x + 2 = \pm 1$
$x = -2 \pm 1$
$x = -3, -1$

$g(0) = -3(0+2)^2 + 3$
$g(0) = -3(4) + 3 = -9$

e. Sketch the graph



f. Find the intervals over which *f* is increasing and decreasing.

Increasing $(-\infty, -2)$
Decreasing $(-2, \infty)$

5. Given: $f(x) = x^2 + 2x - 1$

a. Find the coordinates of the vertex. Is it a maximum or a minimum?

$$x = \frac{-2}{2(1)} = -1$$
$$f(-1) = (-1)^2 + 2(-1) - 1$$
$$f(-1) = 1 - 2 - 1 = -2$$
$$(-1, -2) \text{ minimum}$$

b. Find the equation of the axis of symmetry.

$x = -1$

c. Find the domain and range.

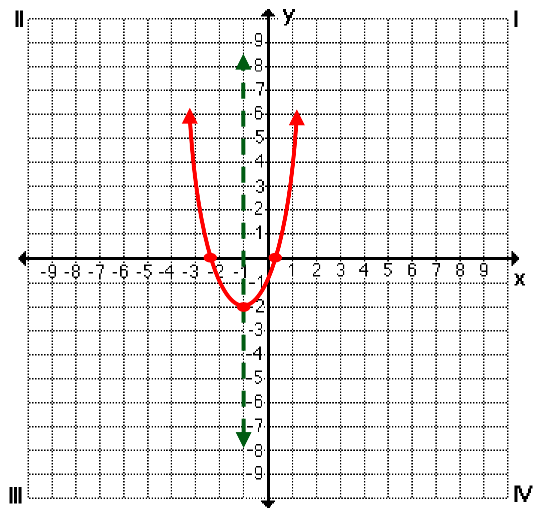
Domain: $(-\infty, \infty)$
Range: $[-2, \infty)$

d. Find the *x*- and *y*-intercepts.

$$0 = x^2 + 2x - 1$$
$$x = \frac{-2 \pm \sqrt{4+4}}{2}$$
$$x = \frac{-2 \pm 2\sqrt{2}}{2}$$
$$x = -1 \pm \sqrt{2}$$
$$x = -2.41, 0.41$$

$$f(0) = 0^2 + 2(0) - 1 = -1$$

e. Sketch the graph



f. Find the intervals over which *f* is increasing and decreasing.

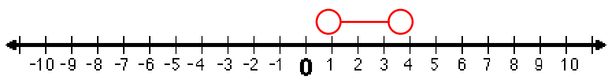
Increasing $(-1, \infty)$
Decreasing $(-\infty, -1)$

Solving Quadratic Inequalities

Examples

7. Solve the inequality. $x^2 - 5x < -4$

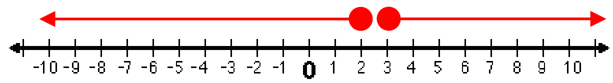
$$x^2 - 5x + 4 < 0$$
$$(x-4)(x-1) < 0$$



$$(1, 4)$$

8. Solve the inequality. $x^2 - 5x + 6 \geq 0$

$$(x-3)(x-2) \geq 0$$



$$(-\infty, 2] \cup [3, \infty)$$

Modeling with Quadratic Functions

Height of an object t seconds after it is dropped from an initial height of h_0 feet	$h(t) = h_0 - 16t^2$

Examples:

9. A farmer wants to enclose a rectangular field along a river on three sides. If 2,400 feet of fencing is to be used, what dimensions will maximize the enclosed area?

$$A = lw$$
$$P = 2l + w$$
$$2400 = 2l + w$$
$$w = 2400 - 2l$$
$$A = l(2400 - 2l)$$
$$A = 2400l - 2l^2$$
$$l = \frac{-2400}{2(-2)} = \frac{-2400}{-4} = 600$$
length = 600 ft

width = 1200 ft

10. A ball is dropped off a 196-ft high building. When will the ball hit the ground?

$$h(t) = -16t^2 + 196$$
$$0 = -4(4t^2 - 49)$$
$$4t^2 - 49 = 0$$
$$(2t+7)(2t-7) = 0$$
$$t = \pm\frac{7}{2}$$
$t = 3.5$ seconds