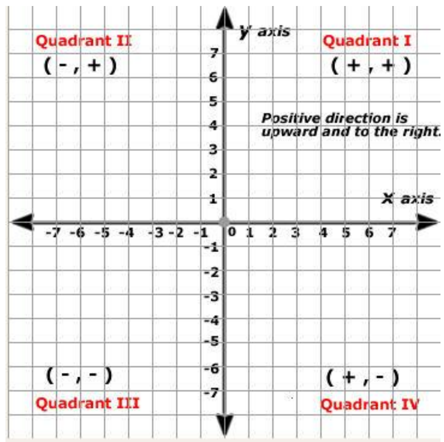
Name: ______________________________

5.1 Cartesian Coordinate System

Essential Question(s):

- **How do you use symmetry as an aid in graphing?**



Ordered Pair	**A pair of numbers for which the order is important.** **(x, y); (Domain, Range); (Abscissa, Ordinate)**
Solution (to an equation in two variables)	**An ordered pair of numbers (x, y) that makes the equation true.**

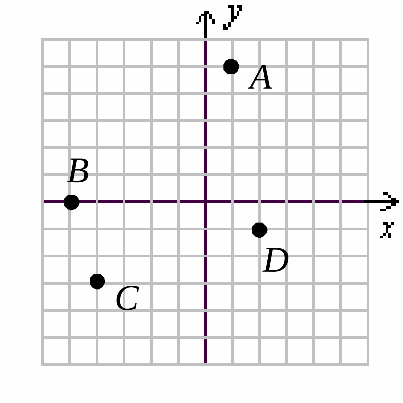
Reflections and Symmetry	
***x*-axis**	**Reflection:** (a,b) → $(a,-b)$ **Symmetry Test:** replace *y* with –*y* then simplify
***y*-axis**	**Reflection:** (a,b) → $(-a,b)$ **Symmetry Test:** replace *x* with –*x* then simplify
origin	**Reflection:** (a,b) → $(-a,-b)$ **Symmetry Test:** replace *x* with –*x* AND *y* with –*y* then simplify

1. Find the coordinates of points *A*, *B*, *C*, and *D*.

A = (1, 5)
B = (–5, 0)
C = (–4, –3)
D = (2, –1)

2. Reflect *A*, *B*, *C*, and *D* through the ***y*-axis** and give the coordinates of the reflected points

A' = (–1, 5)
B' = (5, 0)
C' = (4, –3)
D' = (–2, –1)



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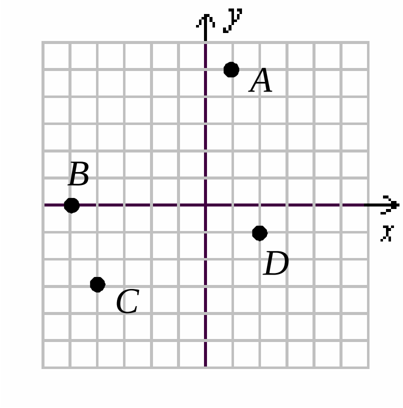
3. Reflect *A*, *B*, *C*, and *D* through the ***x*-axis** and give the coordinates of the reflected points.

A' = (1, –5)
B' = (–5, 0)
C' = (–4, 3)
D' = (2, 1)



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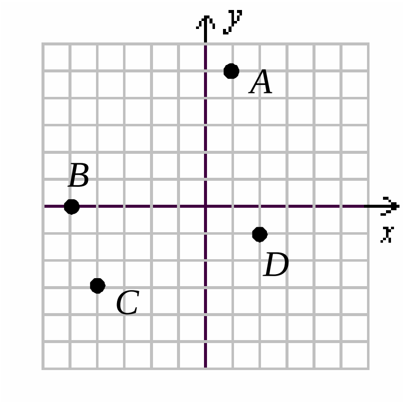
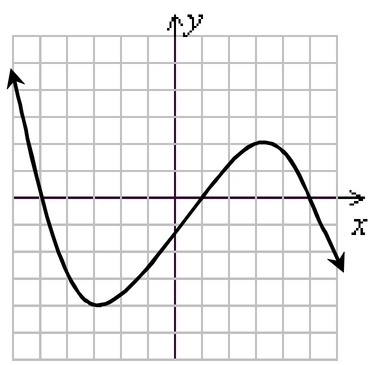
4. Reflect *A*, *B*, *C*, and *D* through the **origin** and give the coordinates of the reflected points.

A = (-1, -5)
B = (5, 0)
C = (4, 3)
D = (-2, 1)





5. Use the graph to estimate to the nearest integer the missing coordiante(s) of the point.

a. (–3, ?)

–4

b. (?, 0)

–5, 1, and 5

6. Test the equation for symmetry with respect to the *x*-axis, the *y*-axis, and the origin.

$x^2 + xy^2 + x = 9$

x – axis:

$x^2 + x(-y)^2 + x = 9$

$x^2 + xy^2 + x = 9$

yes

y – axis:

$(-x)^2 + -xy^2 + -x = 9$

$x^2 - xy^2 - x = 9$

no

origin:

$(-x)^2 + -x(-y)^2 + -x = 9$

$x^2 - xy^2 - x = 9$

no

Symmetric with respect to the *x*-axis

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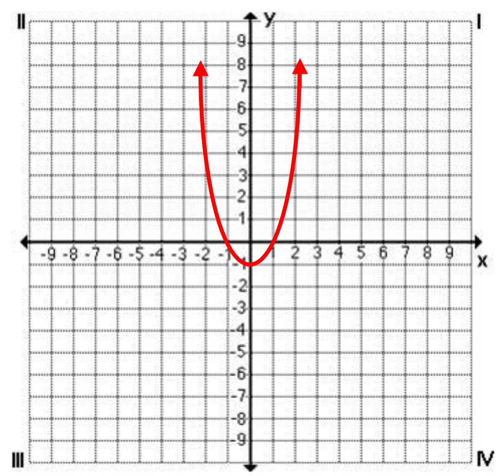
7. Test the equation for symmetry with respect to the *x*-axis, the *y*-axis, and the origin. Sketch the graph of the equation.

$y + 1 = x^2$



x – axis:

$-y + 1 = x^2$

No

y – axis:

$y + 1 = (-x)^2$

$y + 1 = x^2$

Yes

origin:

$-y + 1 = (-x)^2$

$-y + 1 = x^2$

no

Symmetric with respect to the *y*-axis

5.2 Distance in the Plane

Essential Question(s):

- **How do you find the distance between two points?**
- **How do you find the midpoint of a line segment?**
- **How do you write the equation of a circle?**

Distance Formula
$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

1. Find the distance between (–6, 7) and (3, –5).

$$D=\sqrt{(-6-3)^2+[7-(-5)]^2}$$
$$D=\sqrt{(-9)^2+12^2}$$
$$D=\sqrt{81+144}$$
$$D=\sqrt{225}$$
$$D=15$$

2. Find x such that (x, 5) is 10 units from (–2, 11)

$$10=\sqrt{[x-(-2)]^2+(5-11)^2}$$
$$10=\sqrt{(x+2)^2+(-6)^2}$$
$$10=\sqrt{(x+2)^2+36}$$
$$10^2=\left(\sqrt{(x+2)^2+36}\right)^2$$
$$100=(x+2)^2+36$$
$$64=(x+2)^2$$
$$\sqrt{64}=\sqrt{(x+2)^2}$$
$$\pm 8=x+2$$
$$-2\pm 8=x$$
$$-10,6=x$$

Midpoint Formula
$\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$

3. Find the midpoint of the line segment with endpoints (5, 8) and (1, 4).

$$\left(\frac{5+1}{2},\frac{8+4}{2}\right)$$
$$\left(\frac{6}{2},\frac{12}{2}\right)$$
$$(3,6)$$

4. The midpoint of the line segment with endpoints (6, 1) and (b_1, b_2) is (3, 4). Find b_1 and b_2.

$$\frac{6+b_1}{2}=3 \qquad \frac{1+b_2}{2}=4$$
$$6+b_1=6 \qquad 1+b_2=8$$
$$b_1=0 \qquad b_2=7$$

Equations of a Circle	
Standard Form	$(x-h)^2+(y-k)^2=r^2$ Where (h,k) is the center and *r* is the radius
General Form	$x^2+y^2+Dx+Ey+F=0$ Where *D*, *E* and *F* are real numbers

5. Write the equation of a circle with the indicated center and radius.

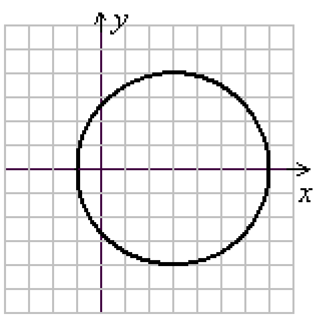
$$C=(3,-2), r=3$$

$$(x-3)^2+(y+2)^2=3^2$$
$$(x-3)^2+(y+2)^2=9$$

6. Write an equation for the set of all points that are one unit from (0, -1).

$$x^2+(y+1)^2=1^2$$
$$x^2+(y+1)^2=1$$

7. Write the equation of the circle.



$(x-3)^2+y^2=16$

8. Find the center and radius of the circle.

$(x-6)^2+(y-8)^2=100.$

Center (6, 8) and radius 10

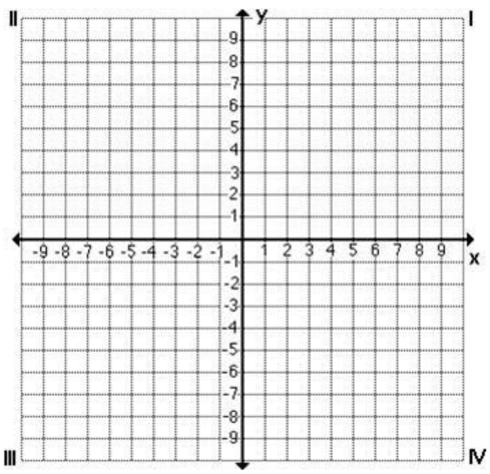
9. Graph the circle by finding the center and radius.

$$x^2 + 4x + y^2 = 0$$

$$\left(x^2 + 4x + 4\right) + y^2 = 0 + 4$$
$$(x+2)^2 + y^2 = 4$$

Center: $(-2, 0)$

Radius: 2



10. Write the given equation of a circle in standard form. Then find the center and radius.

$$x^2 + y^2 - 8x + 6y - 24 = 0$$

$$x^2 - 8x + y^2 + 6y = 24$$
$$x^2 - 8x + 16 + y^2 + 6y + 9 = 24 + 16 + 9$$
$$(x-4)^2 + (y+3)^2 = 49$$

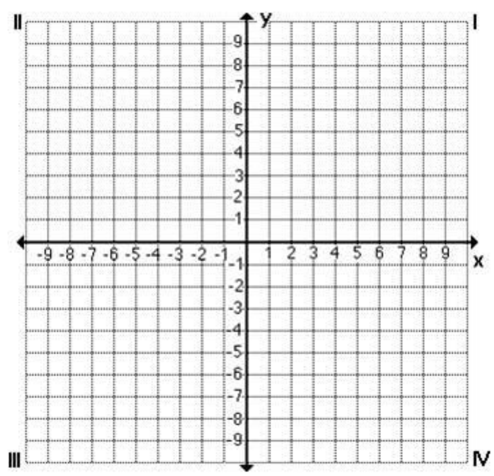
Center: $(4, -3)$

Radius: 7

11. Find the equation of circle with the given center whose graph passes through the given points.

Center: (-5, 4), point on the circle: (2, -3)

$$(2+5)^2 + (-3-4)^2 = r^2$$
$$7^2 + (-7)^2 = r^2$$
$$49 + 49 = r^2$$
$$98 = r^2$$
$$(x+5)^2 + (y-4)^2 = 98$$



5.3 Equations of a Line

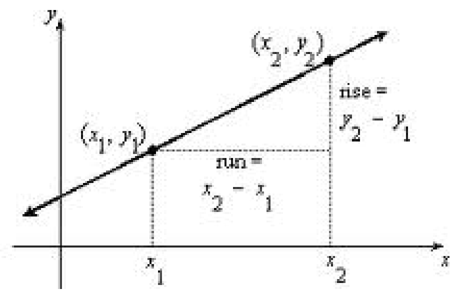
Essential Question(s):

- **How do you find the slope of a line?**
- **How do you find the equation of a line?**

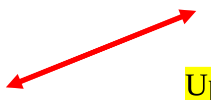
Slope

The slope m of a nonvertical line is the ratio of the vertical change (the *rise*) to the horizontal change (the *run*) between any two points on the line

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{rise}{run} = \text{rate of change}$$

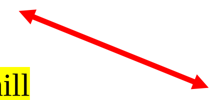


Positive Slope



Uphill

Negative Slope



Downhill

Special Cases

Vertical Lines

Undefined or No Slope

$$m = \frac{6+2}{3-3} = \frac{8}{0} = \varnothing$$

Horizontal Lines

Zero Slope

$$m = \frac{4-4}{-2-5} = \frac{0}{-7} = 0$$

Examples

Find the slope of the line passing through the given points.

1. $(-7,5),\ (4,-2)$

$$m = \frac{5-(-2)}{-7-4} = \frac{7}{-11}$$

2. $(3,5),\ (3,2)$

$$m = \frac{5-2}{3-3} = \frac{3}{0} = \text{undefined}$$

Vertical line

3. $(5,5),\ (-4,5)$

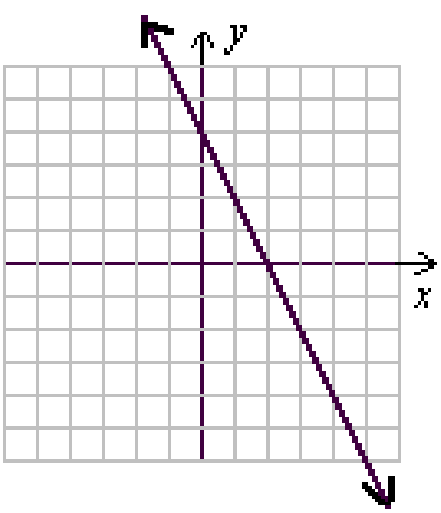
$$m = \frac{5-5}{5-(-4)} = \frac{0}{9} = 0$$

Horizontal line

Different Forms of Linear Equations	
Slope-intercept Form	A linear equation written in the form $\boldsymbol{y = mx + b}$ or $f(x) = mx + b$ • m is the slope • b is the y-intercept. • Best form for graphing.
Standard form	A linear equation written in the form $\boldsymbol{Ax + By = C}$, • A, B and C are integers • A is positive • m is $\frac{-A}{B}$
Point-slope Form	A linear equation written in the form $(y - y_1) = m(x - x_1)$, • m is the slope • (x_1, y_1) is a coordinate on the line
Vertical Line	$x = a$ where a is the x-intercept Vertical lines have undefined or no slope
Horizontal Line	$y = b$ where b is the y-intercept Horizontal lines have zero slope

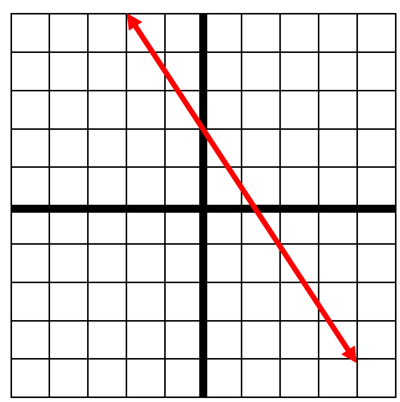
Examples

4. Find the equation in *standard form* of the line.



$$y = -2x + 4$$
$$2x + y = 4$$

5. Graph the line $3x + 2y = 6$.



$$2y = -3x + 6$$
$$y = \frac{-3}{2} + 3$$

6. Given the equation $3x + 2y = 6$, find the slope, if it exists.

$-\frac{3}{2}$

7. Given the equation, $y = -3$, find the slope, if it exists.

0

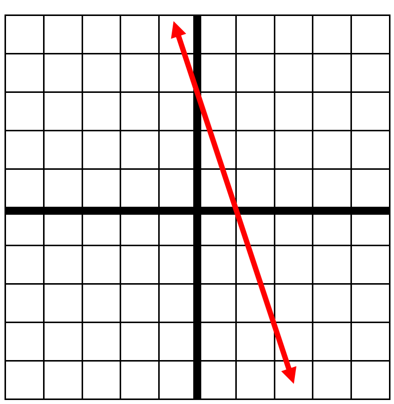
8. Find the equation of the line with slope $\frac{2}{3}$ and y-intercept 8. Write the equation in *standard form.*

$$y = \frac{2}{3}x + 8$$
$$-3\left(-\frac{2}{3}x + y = 8\right)$$
$$2x - 3y = -24$$

9. Write the equation of the line that passes through point (0, 1) with slope $\frac{3}{5}$. Give your answer in the *slope-intercept form*.

$y = \frac{3}{5}x + 1$

10. Sketch a graph of the line that contains the point (0, 3) and has slope –3. Then write the equation of the line in the slope intercept form.



$y = -3x + 3.$

11. Write the equation of the line passing through (–4, –7) and (3, 0). Write your answer in the slope-intercept form.

$$m = \frac{-7-0}{-4-3} = \frac{-7}{-7} = 1$$
$$y = 1(x-3)$$
$$y = x-3$$

Parallel and Perpendicular Lines	
Parallel Lines	Parallel lines have equal slopes Examples: $m = 2$ and $m_{\parallel} = 2$ $m = 3/4$ and $m_{\parallel} = 3/4$
Perpendicular Lines	Perpendicular Lines have opposite reciprocal slopes (flip the fraction and change the sign) Examples: $m = 2$ and $m_{\perp} = -1/2$ $m = 3/4$ and $m_{\perp} = -4/3$

12. Write an equation of the line passing through (–4, –7), and *parallel* to $y = 2x + 5$. Write your answer in *standard form*.

$$y + 7 = 2(x + 4)$$
$$y + 7 = x + 8$$
$$y = 2x + 1$$
$$-2x + y = 1$$
$$-1(-2x + y = 1)$$
$$2x - y = -1$$

13. Write an equation of the line passing through (–8, –3), and *perpendicular* to $y = \frac{1}{4}x + 2$. Write your answer in standard form.

$$y + 3 = -4(x + 8)$$
$$y + 3 = -4x - 32$$
$$y = -4x - 35$$
$$4x + y = -35$$

5.4 Linear Equations and Models

Essential Question(s):

- **How do you find the line of best fit?**

Mathematical Model	**Mathematical representation (an equation/graph) of a real-world problem**
Linearly related variables	**Variables related by a linear equation**
Rate of change	**The slope of a linear equation**
Regression analysis (Curve fitting)	**The process of finding a function to model a set of data points**
Scatter Plot	**The graph of the points in a data set**
Regression line	**The line of best fit for a set of data points** **Sometimes called the Least-squares regression line**
Interpolation	**Using the regression line to approximate points located within the range of the data set**
Extrapolation	**Using the regression line to approximate points located outside of the range of the data set**

Use the following to answer questions 1-2:
The Number Two Plumbing Co. charges $35 per hour plus a fixed service call charge of $45.

1. Write an equation that will allow you to compute the total bill for any number of hours, *x*, that it takes to complete a job.

   $C = 35x + 45$

2. If the bill comes to $120.25, how many hours did the job take?

   $120.25 = 35x + 45$
   $75.25 = 35x$
   $2.15 = x$

   2.15 hours

Use the following to answer questions 3-6:
A driver going down a straight highway is traveling at 70 ft/sec on cruise control when he begins accelerating at a rate of 4.2 ft/sec^2. The velocity of the car in ft/sec is given by the function $V = 4.2t + 70$, where *t* is in seconds.

3. Interpret the meaning of the slope of this model.

   Every second the velocity is increasing by 4.2 ft/sec.

4. What is the effect of a 1 second increase in time traveled?

   The velocity increases by 4.2 ft/sec.

5. Determine the velocity of the car after 10.4 seconds.

   $V = 4.2(10.4) + 70$
   $V = 43.68 + 70$
   $V = 113.68$
   113.68 ft/sec

6. If the car is traveling at 100 ft/sec, for how long did it accelerate? (Round to the nearest tenth of a second.)

   $100 = 4.2t + 70$
   $30 = 4.2t$
   $7.1 = t$
   7.1 seconds

Regression Analysis on TI Graphing Calculator

1. Enter all *x*- and *y*-values into a list: Stat → Edit
   → Enter *x*-values into list 1 (L1),
   →Enter *y*-values into list 2 (L2)
2. Calculate and store the regression equation: Stat→ Calc → LinReg (ax+b) 2nd 1 , 2nd 2 , VARS →Y-Vars→function→Y1→Enter
3. Turn on the scatter plot: 2nd→Y = →Enter→On, Type: Scatter, Xlist:L1,Ylist:L2
4. Graph: Zoom→9:ZStat→Enter

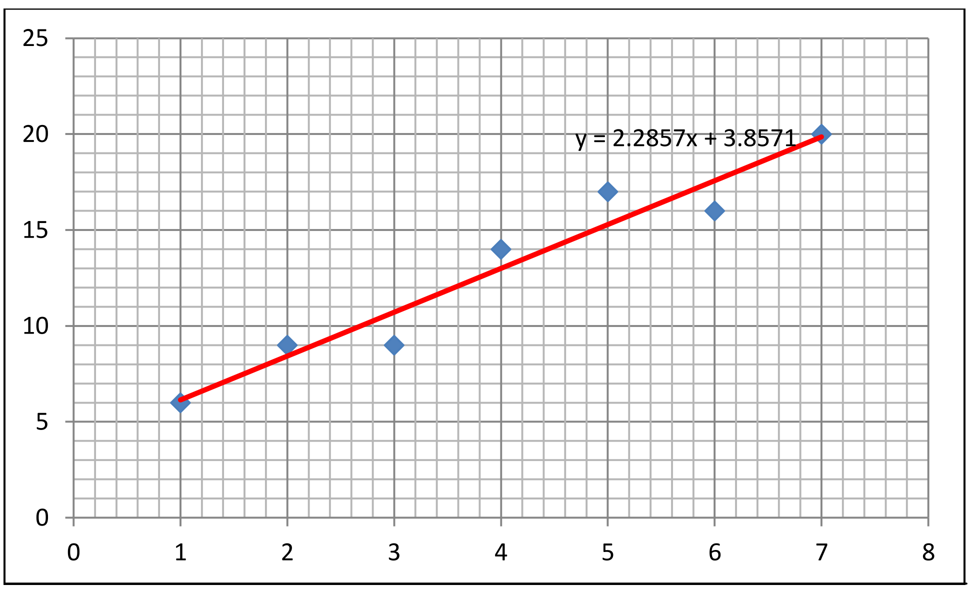
Use the following ordered pairs to answer questions 7-10:

(4, 14), (1, 6), (3, 9), (2, 9), (5, 17), (7, 20), (6, 16)

7. Find the linear regression for the data. (Round all values to the nearest hundredth.)

$y = 2.28x + 3.85$

8. Plot the data and the model on the same axes.



9. Use the model to estimate *y* when *x* = 3.5.

$y = 2.28(3.5) + 3.85$

$y = 11.83$

10. Use the model to estimate *y* when *x* = 20.

$y = 2.28(20) + 3.85$

$y = 49.45$