Name: ___________________________

4.1 Complex Numbers

Essential Question(s):

- **How do you add, subtract, multiply, and divide complex numbers?**

Vocabulary:

Standard Form of a Complex Number	$a+bi$
Zero	$0=0+0i$
Conjugate	$a-bi$

Anatomy of a Complex Number:

$$a+bi$$

a: Real Part

bi: Imaginary Part

Properties of Equality and Basic Operations:

Equality	$a+bi=c+di$ iff $a=c$ and $b=d$ Set reals=reals & imaginaries=imaginaries
Addition	$(a+bi)+(c+di)=(a+c)+(b+d)i$ Add reals to reals & imaginaries to imaginaries
Multiplication	$(a+bi)(c+di)=(ac-bd)+(ad+bc)i$ FOIL
Additive Identity	$0+0i$
Additive Inverse	$-a-bi$
Multiplicative Identity	1
Principle Square Root of a Negative Number	$\sqrt{-a}=i\sqrt{a}$

Cyclic Properties of *i*:

$i=\sqrt{-1}$ → $i^2=-1$ → $i^3=-\sqrt{-1}$ → $i^4=1$ → $i=\sqrt{-1}$

Examples:

1. For the complex number 4 – 2*i*, find…

a. the real part 4

b. the imaginary part -2*i*

c. the conjugate 4 + 2*i*

2. Add. Write the result in standard form.

(10 – 10*i*) + (1 + 7*i*)

11 – 3i

3. Subtract. Write the result in standard form.

(–4 – 8*i*) – (–1 – 7*i*)

$$-4-8i+1+7i$$
$$-3-i$$

4. Multiply. Write the result in standard form.

(5 – *i*)(4 + 3*i*)

23 + 11i

5. Evaluate. Write your answer in standard form.

$$\sqrt{-5}\cdot\sqrt{-80}$$

$$i\sqrt{5}\cdot 4i\sqrt{5}$$
$$4i^2(5)$$
$$-4(5)$$
$$-20$$

6. Evaluate. Write your answer in standard form.

$$\sqrt{5}\cdot\sqrt{-80}$$

$$\sqrt{5}\cdot 4i\sqrt{5}$$
$$4i(5)$$
$$20i$$

7. Divide. Write your answer in standard form.

$$\frac{1+7i}{2i}$$

$$\left(\frac{1+7i}{2i}\right)\left(\frac{i}{i}\right)$$
$$\frac{1i+7i^2}{2i^2}$$
$$\frac{-7+i}{-2}$$
$$\frac{7}{2}-\frac{i}{2}$$

8. Divide and write your answer in standard form.

$$\frac{-2+4i}{1+3i}$$

$$\left(\frac{-2+4i}{1+3i}\right)\left(\frac{1-3i}{1-3i}\right)$$
$$\frac{-2+6i+4i-12i^2}{1-9i^2}$$
$$\frac{10+10i}{10}$$
$$1+i$$

9. Solve for *x* and *y*.

$$(x+4)+(y-4)i=2+3i$$

Reals

$x+4=2$

$x=-2$

Imaginaries

$y-4=3$

$y=7$

10. Solve for *x* and *y*.

$$\frac{(4+x)+(y+3)i}{2-i}=3+i$$
$$(4+x)+(y+3)i=(3+i)(2-i)$$
$$(4+x)+(y+3)i=6-3i+2i-i^2$$
$$(4+x)+(y+3)i=7-1i$$

Reals

$4+x=7$

$x=3$

Imaginaries

$y+3=-1$

$y=-4$

4.2 Quadratic Equations and Applications

Essential Question(s):

- **What are the ways to solve quadratic equations?**
- **How do you solve quadratic word problems?**

Vocabulary:

Standard Form of a Quadratic Equation	$ax^2 + bx + c = 0$
Real root	a real number solution of an equation
Imaginary root	an imaginary number solution of an equation

Methods for solving quadratic equations:

I. Using Factoring and the Zero Property

Zero Property	If *m* and *n* are complex numbers, $mn = 0 \text{ iff } m = 0,\ n = 0, \text{ or both}$

Examples: Solve by factoring.

1. $4x^2 - 20x = -25$

$$4x^2 - 20x + 25 = 0$$
$$(2x-5)^2 = 0$$
$$x = \frac{5}{2}$$

2. $7x^2 - 6 = -19x$

$$7x^2 + 19x - 6 = 0$$
$$(7x-2)(x+3) = 0$$
$$x = \frac{2}{7}, -3$$

II. Using the Square Root Property

Square Root Property	If $A^2 = C$, then $A = \pm\sqrt{C}$

Examples: Solve using the square root property.

3. $(x-4)^2 = 100$

$$x - 4 = \pm 10$$
$$x = -6,\ 14$$

4. $(x-3)^2 - 7 = 0$

$$x - 3 = \pm\sqrt{7}$$
$$x = 3 \pm \sqrt{7}$$

III. Completing the Square:

Steps to Solve
1. Transform the equation so that the constant term c is alone on the right side 2. If a is not equal to 1, then divide both sides by a. 3. Add $\left(\frac{b}{2}\right)^2$ to **BOTH** sides. 4. Factor the left side. 5. Use the square root property 6. Simplify.

Examples: Solve by completing the square.

5. $x^2-6x-2=0$

$$x^2-6x=2$$
$$x^2-6x+9=2+9$$
$$(x-3)^2=11$$
$$x-3=\pm\sqrt{11}$$
$$x=3\pm\sqrt{11}$$

6. $x^2+2x-8=0$

$$x^2+2x+1=8+1$$
$$(x+1)^2=9$$
$$x+1=\pm\sqrt{9}$$
$$x=-1\pm3$$
$$x=-4,2$$

7. $x^2+x+10=0$

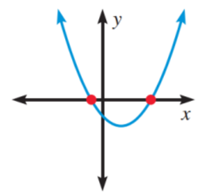
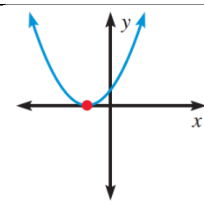
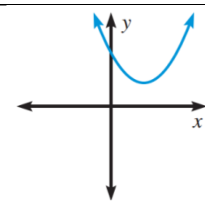
$$x^2+x+\frac{1}{4}=-10+\frac{1}{4}$$
$$\left(x+\frac{1}{2}\right)^2=\frac{-39}{4}$$
$$x+\frac{1}{2}=\pm\frac{\sqrt{-39}}{2}$$
$$x=-\frac{1}{2}\pm\frac{-\sqrt{39}}{2}$$

8. $4x^2+2x-3=0$

$$x^2+\frac{1}{2}x-\frac{3}{4}=0$$
$$x^2+\frac{1}{2}x=\frac{3}{4}$$
$$x^2+\frac{1}{2}x+\frac{1}{16}=\frac{3}{4}+\frac{1}{16}$$
$$\left(x+\frac{1}{4}\right)^2=\frac{13}{16}$$
$$x+\frac{1}{4}=\pm\frac{\sqrt{13}}{4}$$
$$x=-\frac{1}{4}\pm\frac{\sqrt{13}}{4}$$

IV. Using the Quadratic Formula:

Quadratic Formula	$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ ***Equation must be in standard form***
Discriminant	$b^2 - 4ac$

The Discriminant		
Value of the Discriminant:	**Number of Roots:**	**Graphical Representation:**
$b^2 - 4ac > 0$ Discriminant is positive.	Two Real Roots • If the discriminant is a **perfect square**→the roots are **rational** If the discriminant is **not a perfect square**→the roots are **irrational**	 Two *x*-intercepts
$b^2 - 4ac = 0$ Discriminant is zero.	One Real Root	 One *x*-intercept
$b^2 - 4ac < 0$ Discriminant is negative.	Two Imaginary Conjugate Roots	 No *x*-intercept

Examples: Solve using the quadratic formula.

7. $2x^2 + 6x = 11$

$2x^2 + 6x - 11 = 0$

$$x = \frac{-6 \pm \sqrt{6^2 - 4(2)(-11)}}{2(2)}$$

$$= \frac{-6 \pm \sqrt{124}}{4}$$

$$= \frac{-6 \pm 2\sqrt{31}}{4}$$

$$= \frac{-3 \pm \sqrt{31}}{2}$$

8. $x^2 = -x - 11$

$x^2 = -x - 11$

$x^2 + x + 11 = 0$

$$x = \frac{-1 \pm \sqrt{1^2 - 4(1)(11)}}{2(1)}$$

$$= \frac{-1 \pm \sqrt{1 - 44}}{2}$$

$$= \frac{-1 \pm i\sqrt{43}}{2}$$

Find the number of real roots of each quadratic equation.

9. $3x^2 - 6x + 5 = 0$

$(-6)^2 - 4(3)(5)$

$36 - 60$

$-24 < 0$

two imaginary conjugates

10. $3x^2 - 6x + 1 = 0$

$3x^2 - 6x + 1 = 0$

$(-6)^2 - 4(3)(1)$

$36 - 12$

$24 > 0$

24 is not a perfect square

two unequal real irrational roots

11. $3x^2 - 6x + 3 = 0$

$3x^2 - 6x + 3 = 0$

$(-6)^2 - 4(3)(3)$

$36 - 36 = 0$

one real double root

V. Application Examples:

12. The product of two consecutive positive even integers is 168. Find the integers.

$x(x+2) = 168$

$x^2 + 2x - 168 = 0$

$(x+14)(x-12) = 0$

$x = \cancel{-14},\ 12$

$x = 12$

$x + 2 = 14$

13. One positive number is 4 less than twice another positive number and their product is 96. Set-up an algebraic equation and solve it to find the two numbers.

$x \bullet (2x - 4) = 96$

$2x^2 - 4x - 96 = 0$

$x^2 - 2x - 48 = 0$

$(x+6)(x-8) = 0$

$x = \cancel{-6}, 8$

$x = 8$

$2x - 4 = 12$

14. Two technicians can complete a mailing in 3 hours when working together. Alone, one can complete the mailing 2 hours faster than the other. How long will it take each person to complete the mailing alone? Compute the answers to two decimal places.

	WR	T	WD
Tech 1	$\frac{1}{x}$	3	$\frac{3}{x}$
Tech 2	$\frac{1}{x+2}$	3	$\frac{3}{x+2}$

$$\frac{3}{x}+\frac{3}{x+2}=1$$
$$3(x+2)+3x=x(x+2)$$
$$3x+6+3x=x^2+2x$$
$$0=x^2-4x-6$$
$$x=\frac{4\pm\sqrt{16-4(-6)}}{2}$$
$$x=\frac{4\pm 6.32}{2}$$
$$x=2\pm 3.16$$
$$x=5.16,-1.16$$
5.16 hours and 7.16 hours

15. A speedboat takes 3 hours longer to go 60 miles up a river than to return. If the boat cruises at 15 miles per hour in still water, what is the rate of the current?

	R	T	D
Upstream	15 – *x*	t + 3	60
Downstream	15 + *x*	t	60

Upstream: $(15-x)(t+3)=60$

$$t=\frac{60}{15-x}-3$$

Downstream: $(15+x)t=60$

$$t=\frac{60}{15+x}$$

$$\frac{60}{15-x}-3=\frac{60}{15+x}$$
$$60(15+x)-3(15+x)(15-x)=60(15-x)$$
$$3x^2+120x-675=0$$
$$x^2+40x-225=0$$
$$(x+45)(x-5)=0$$
$$x=-45,\ 5$$
$$x=5\text{ mi / h}$$

4.3 Special Equation Solving Techniques

Essential Question(s):

- **How do you solve equations involving radicals and absolute value?**
- **How do you solve equations of the quadratic type?**

I. Equations involving Radicals

When **squaring** both sides of an equation, **extraneous** solutions are introduced. You must **check** all possible solutions in the **original** equation to eliminate the **extraneous** solutions.

Examples:

1. Solve.

$$\sqrt{x-1} = x-3$$

$$\left(\sqrt{x-1}\right)^2 = (x-3)^2$$

$$x-1 = x^2-6x+9$$

$$0 = x^2-7x+10$$

$$0 = (x-5)(x-2)$$

$$x = 5, \not{2}$$

2. Solve.

$$\sqrt{3x+7} - \sqrt{2x-3} = 2$$

$$\sqrt{3x+7} = \sqrt{2x-3}+2$$

$$\left(\sqrt{3x+7}\right)^2 = \left(\sqrt{2x-3}+2\right)^2$$

$$3x+7 = 2x-3+4\sqrt{2x-3}+4$$

$$x+6 = 4\sqrt{2x-3}$$

$$(x+6)^2 = \left(4\sqrt{2x-3}\right)^2$$

$$x^2+12x+36 = 16(2x-3)$$

$$x^2-20x+84 = 0$$

$$(x-6)(x-14) = 0$$

$$x = 6,\ 14$$

II. Equations involving Absolute Value

Squaring both sides eliminates the need to consider both cases. However, you must still **check** all possible solutions in the **original** equation to eliminate the **extraneous** solutions.
This technique only applies to **equations**. It will NOT work properly with inequalities.

Examples:

3. Solve.

$$|x+5|=1-3x$$

$$|x+5|^2=(1-3x)^2$$
$$x^2+10x+25=1-6x+9x^2$$
$$0=8x^2-16x-24$$
$$0=x^2-2x-3$$
$$0=(x-3)(x+1)$$
$$x=\cancel{3},\ -1$$

4. Solve.

$$|6x-1|=x-6$$

$$|6x-1|^2=(x-6)^2$$
$$36x^2-12x+1=x^2-12x+36$$
$$35x^2=35$$
$$x^2=1$$
$$x=\cancel{-1},\ \cancel{1}$$
$$\varnothing$$

III. Equations involving the Quadratic Form

Use u-substitution.

Examples:

5. Solve.

$$5x^{\frac{2}{3}}-13x^{\frac{1}{3}}-6=0$$

$$\text{Let } u=x^{\frac{1}{3}}$$
$$5u^2-13u-6=0$$
$$(5u+2)(u-3)=0$$
$$u=-\frac{2}{5},\ 3$$
$$x=u^3$$
$$x=-\frac{8}{125},\ 27$$

6. Solve.

$$x+5\sqrt{x}-24=0$$

$$\text{Let } u=x^{\frac{1}{2}}$$
$$u^2+5u-24=0$$
$$(u+8)(u-3)=0$$
$$u=-8,\ 3$$
$$x=u^2$$
$$x=\cancel{64},\ 9$$