Name: ____________________________

3.1 Linear Equations and Applications

Essential Question(s):

- **How do you solve linear equations?**
- **How do you solve linear word problems?**

Vocabulary:

Algebraic Equation	Formed by placing an equal sign between two algebraic expressions Can be solved
Algebraic Expression	Part of an algebraic equation. Can only be *simplified*. **Caution:** Methods for solving and methods for simplifying are not always the same.
Domain (replacement set)	The set of numbers that are permitted to replace the variable The set of x-values
Solution or Root	Each element in the domain of the variable that makes the equation true
Solving an Equation	Finding the complete solution set for the equation
Standard Form of a Linear Equation	$Ax + By = C$, where $A \neq 0$

Properties of Equality: If a, b, and c are any real numbers and a = b, then

Addition Property of Equality	$a+c=b+c$	**Add the same value to BOTH sides of an equation**
Subtraction Property of Equality	$a-c=b-c$	**Subtract the same value from BOTH sides of an equation**
Multiplication Property of Equality	$ca=cb$	**Multiply the same value on BOTH sides of an equation**
Division Property of Equality	$\frac{a}{c}=\frac{b}{c},\ c\neq 0$	**Divide by the same value BOTH sides of an equation**
Substitution Property		**You may replace an expression with an equivalent expression without changing its value.** **"plugging into" an equation**

Examples:

1. Solve. $4(x + 5) + 7x = 8x + 11$

$$\begin{aligned} 4x+20+7x&=8x+11 \\ 11x+20&=8x+11 \\ 3x&=-9 \\ x&=-3 \end{aligned}$$

2. Solve. $\frac{4x-3}{5}-6=\frac{x}{2}$

$$\begin{aligned} 10\left(\frac{4x-3}{5}-6\right)&=10\left(\frac{x}{2}\right) \\ 2(4x-3)-60&=5x \\ 8x-6-60&=5x \\ 8x-66&=5x \\ 3x&=66 \\ x&=22 \end{aligned}$$

When we simplified rational *expressions*, we needed to make sure every denominator was the same before adding or subtracting. **BUT** in rational *equations* we can **eliminate the fractions** completely by multiplying by the LCD.

3. Solve for *r*. $m=n+(p-5)r$

$$\begin{aligned} m&=n+(p-5)r \\ m-n&=(p-5)r \\ \frac{m-n}{p-5}&=r \end{aligned}$$

4. Solve for *s*. $\frac{1}{r}=\frac{1}{s}+\frac{1}{t}$

$$\begin{aligned} \frac{1}{r}&=\frac{1}{s}+\frac{1}{t} \\ \frac{1}{r}-\frac{1}{t}&=\frac{1}{s} \\ \frac{t-r}{rt}&=\frac{1}{s} \\ \frac{rt}{t-r}&=s \end{aligned}$$

5. Find three consecutive odd integers such that 3 times their sum is 5 more than 8 times the middle one.

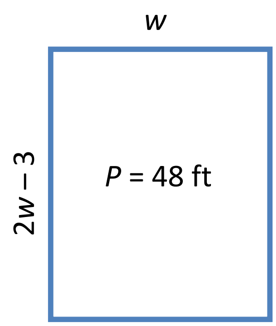
$$
\begin{aligned}
3[x+(x+2)+(x+4)] &= 8(x+2)+5 \\
3[3x+6] &= 8x+16+5 \\
9x+18 &= 8x+21 \\
x+18 &= 21 \\
x &= 3 \\
&3, 5, 7
\end{aligned}
$$

6. Find three consecutive odd integers such that the sum of the second, twice the first, and three times the third is 152.

$$
\begin{aligned}
2x+(x+2)+3(x+4) &= 152 \\
6x+14 &= 152 \\
6x &= 138 \\
x &= 23 \\
&23, 25, 27
\end{aligned}
$$

6. The length of a rectangle is 3 ft less than 2 times its width. If the perimeter of the rectangle is 48 ft, find the dimensions of the rectangle.

$$
\begin{aligned}
w &= width \\
2w-3 &= length \\
P &= 48 \\
P &= 2l+2w \\
48 &= 2(2w-3)+2w \\
48 &= 4w-6+2w \\
54 &= 6w \\
w &= 9 \text{ ft} \\
2w-3 &= 15 \text{ ft}
\end{aligned}
$$



7. How much pure antifreeze must be added to 12 gallons of 20% antifreeze to make a 40% antifreeze solution?

	# of Gal	% Antifreeze	Total Antifreeze
Orig. Sol.	12	20	240
Antifreeze	x	100	100x
New Sol.	12 + x	40	40(12+x)

$$240+100x=40(12+x)$$
$$240+100x=480+40x$$
$$60x=240$$
$$x=4\text{ gal}$$

8. One computer printer can print a company's mailing labels in 40 minutes. A second printer would take 60 minutes to print the labels. How long would it take the two printers, operating together, to print the labels?

	WR	T	WD
Printer 1	$\frac{1}{40}$	x	$\frac{x}{40}$
Printer 2	$\frac{1}{60}$	x	$\frac{x}{60}$

$$\frac{x}{40}+\frac{x}{60}=1$$
$$3x+2x=120$$
$$5x=120$$
$$x=24\text{ min}$$

9. Bill's motorboat can travel 30 mi/h in still water. If the boat can travel 9 miles downstream in the ***same time*** it takes to travel 1 miles upstream, what is the rate of the river's current?

	R	T	D
Upstream	$30-x$	t	1
Downstream	30 + x	t	9

Downstream: $(30+x)t=9$

$$t=\frac{9}{30+x}$$

Upstream: $(30-x)t=1$

$$t=\frac{1}{30-x}$$

$$\frac{9}{30+x}=\frac{1}{30-x}$$

$$270-9x=30+x$$

$$240=10x$$

$$24\text{ mi / h }=x$$

3.2 Linear Inequalities

Essential Question(s):

- **How do you solve linear inequalities?**

Vocabulary:

Inequality Symbols			
>	***"greater than"***	**<**	***"less than"***
$\geq$	***"greater than or equal to"***	$\leq$	***"less than or equal to"***

Trichotomy Property	**For any two real numbers a and b,** $a < b,\ \text{or}\ a > b,\ \text{or}\ a = b$
Interval	**The subset of real numbers that is the solution to an inequality.** • **[ ] denotes a closed interval (endpoints included in the interval). Use closed circles when graphing on the number line.** • **( ] or [ ) denote a half-open interval** • **( ) denotes an open interval (endpoints not included in the interval). Use open circles when graphing on the number line.**
$A \cup B$	**The union of sets A "OR" B. Combines all of set A with all of set B.**
$A \cap B$	**The intersection of sets A "AND" B. Combines what is in common in sets A and B**
Solution Set of an Inequality	**The set of all values of the variable that make the inequality a true statement**
Solving an inequality	**Finding the solution set of the inequality**

Inequality Properties: If a, b, and c are any real numbers,

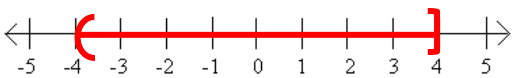
Transitive Property	**If *a* < *b* and *b* < *c*, then *a* < *c***
Addition Property	**If *a* < *b*, then *a* + *c* < *b* + *c*** **Add the same value to BOTH sides of an inequality**
Subtraction Property	**If *a* < *b*, then *a* – *c* < *b* – *c*** **Subtract the same value from BOTH sides of an inequality**
Multiplication Property	**If *a* < *b* and *c* is positive, then *ca* < *cb*** **Multiplying the same POSITIVE value on BOTH sides of an inequality will NOT change the inequality.** **If *a* < *b* and *c* is negative, then *ca* > *cb*** **Multiplying the same NEGATIVE value on BOTH sides of an inequality WILL REVERSE the inequality symbol.**
Division Property	**If *a* < *b* and *c* is positive, then** $\frac{a}{c} < \frac{b}{c}$ **Dividing the same POSITIVE value on BOTH sides of an inequality will NOT change the inequality.** **If *a* < *b* and *c* is negative, then** $\frac{a}{c} > \frac{b}{c}$ **Dividing the same NEGATIVE value on BOTH sides of an inequality WILL REVERSE the inequality symbol.**

Examples:

1. Rewrite in *inequality notation* and graph on a real number line.

(–4, 4]

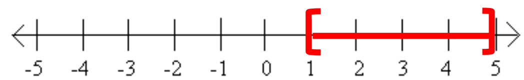
$-4 < x \leq 4$



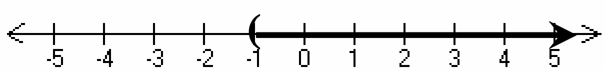
2. Rewrite in *interval notation* and graph on a real number line.

$1 \leq x \leq 5$

[1, 5]



3. Write in *interval notation* and *inequality notation.*



(–1, ∞)

$x > -1$

4. Fill in the blanks with > or < to make the resulting statement true.

$-4 \underline{<} -2$

and

$-4 - 3 \underline{<} -2 - 3$

5. Graph and write as a single interval, if possible.

a. [–3, 6) ∩ [5, 8)

[5, 6]

b. [–3, 6) ∪ [5, 8)

[–3, 8)

6. For what real numbers *x* does the expression represent a real number?

$\sqrt{x-6}$

$x - 6 \geq 0$

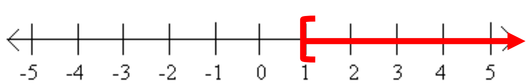
$x \geq 6$

7. Solve and graph.

$$3x - 7 \geq x - 5$$

$$3x - 7 \geq x - 5$$
$$2x \geq 2$$
$$x \geq 1$$

$$[1, \infty)$$

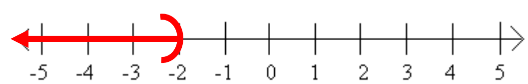


8. Solve and graph.

$$\frac{3y}{7} + \frac{y}{14} < -1$$

$$14\left(\frac{3y}{7} + \frac{y}{14}\right) < 14(-1)$$
$$6y + y < -14$$
$$7y < -14$$
$$y < -2$$

$$(-\infty, -2)$$



9. If *F* is the temperature in degrees Fahrenheit, then the temperature *C* in degrees Celsius is given by the formula $C = \frac{5}{9}(F - 32)$. For what Fahrenheit temperatures will the Celsius temperature be between –5 and 35, inclusive?

$$-5 \leq C \leq 35$$
$$-5 \leq \frac{5}{9}(F - 32) \leq 35$$
$$\left(\frac{9}{5}\right)(-5) \leq \left(\frac{9}{5}\right)\left(\frac{5}{9}(F - 32)\right) \leq \left(\frac{9}{5}\right)(35)$$
$$-9 \leq F - 32 \leq 63$$
$$23 \leq F \leq 95$$

3.3 Absolute Value in Equations and Inequalities

Essential Question(s):

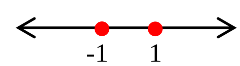
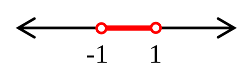
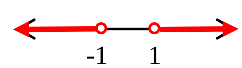
- **How do you solve absolute value equations?**
- **How do you solve absolute value inequalities?**

Steps to Solve

1. Isolate the absolute value.
2. Write absolute value on left side.
3. Determine if "and" or "or."
   - And : $<, \leq$
   - Or : $=, >, \geq$
4. Set up 2 equations.
   - Drop absolute value & solve.
   - Drop absolute value, flip the inequality, take the opposite and solve.

Solutions should ALWAYS be graphed as well as written in inequality AND interval notation

Examples:

	Equation/Inequality	Inequality Notation	Interval Notation	Graph
Equality	$\lvert x \rvert = 1$	$x = 1$ or $x = -1$	$\{-1, 1\}$	
Less Than	$\lvert x \rvert < 1$	$x < 1$ and $x > -1$	$(-1, 1)$	
Greater Than	$\lvert x \rvert > 1$	$x > 1$ or $x < -1$	$(-\infty, -1) \cup (1, \infty)$	

Notes:

- If $\lvert x \rvert$ = negative number or $\lvert x \rvert$<0 or negative number, there is no solution.

  Translation: absolute values **cannot** be negative
- If $\lvert x \rvert$ = 0 or $\lvert x \rvert \leq 0$, there is one solution.
- If $\lvert x \rvert$ > negative number or $\lvert x \rvert \geq 0$, the solution is all real numbers.

  Translation: absolute values are **always** positive

Examples:

Solve. How many solutions does each problem yield?

1. $|y| = 7$

$y = 7$ or $y = -7$

$\{-7, 7\}$

2 sol's

2. $|w| = 0$

$w = 0$ or $w = 0$

$\{0\}$

1 sol

3. $|z| = -12$

No solution

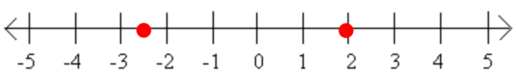
Solve. Solutions should be graphed as well as written in *inequality* and *interval notations*.

4. $|4x + 1| = 9$

$4x + 1 = 9$ or $4x + 1 = -9$

$4x = 8$ $\quad$ $4x = -10$

$x = 2$ $\quad$ $x = -2.5$



5. $|3x + 3| \le 9$

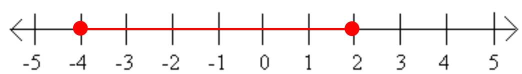
$3x + 3 \le 9$ and $3x + 3 \ge -9$

$3x \le 6$ $\quad$ $3x \ge -12$

$x \le 2$ $\quad$ $x \ge -4$

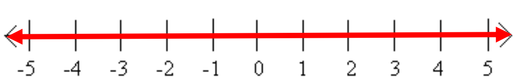
$\{x : -4 \le x \le 2\}$

$[-4, 2]$



6. $|3x - 1| \ge 0$

$(-\infty, \infty)$



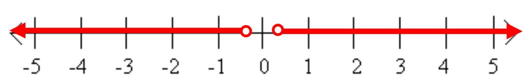
7. $|3x - 1| > 0$

$3x - 1 > 0$ or $3x - 1 < 0$

$3x > 1$ $\quad$ $3x < 1$

$x > \frac{1}{3}$ $\quad$ $x < \frac{1}{3}$

$\left(-\infty, \frac{1}{3}\right) \cup \left(\frac{1}{3}, \infty\right)$



8. $|4x-3|>5$

$4x-3>5$ or $4x-3<-5$

$4x>8 \qquad 4x<-2$

$x>2 \qquad x<-\frac{1}{2}$

$\left(-\infty,-\frac{1}{2}\right)\cup(2,\infty)$

9. $\sqrt{(2x-1)^2}<9 \rightarrow |2x-1|<9$

$2x-1<9$ and $2x-1>-9$

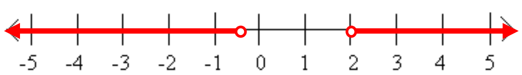
$2x<10 \qquad 2x>-8$

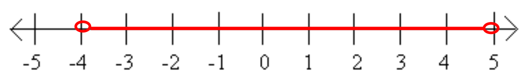
$x<5 \qquad x>-4$

$\{x:-4<x<5\}$

$(-4,5)$

$\sqrt{x^2}=|x|$





10. Solve. $|x+10|=2x+1$

Case 1: $x+10\geq 0$ (that is, $x\geq -10$)

$x+10=2x+1$

$-x=-9$

$x=9$

Case 2: $x+10<0$ (that is, $x<-10$)

$-(x+10)=2x+1$

$-x-10=2x+1$

$-3x=11$

$x=-\frac{11}{3}$

So, $x=9$