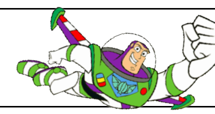
 Name: ____________________________

1.1 Algebra and Real Numbers

Essential Question(s):
- **How do you categorize real numbers?**
- **How do you add, subtract, multiply, and divide real numbers?**

Set	A collection of objects
Element (Member)	Each object in a set **Ex.** $3 \in \{1,2,3\}$ *"3 is an element of the set"*
Finite Set	Elements in the set can be counted
Infinite Set	Elements in the set can be counted without end 
Empty (Null) Set	The set that contains no elements. **Notation:** $\varnothing$ **Note:** The empty set is a subset of any set.
Listing (Roster) Method	Set notation that lists the elements of a set. **Ex:** A = { 1, 2, 3, 4 }
Set Builder (Rule) Notation	Set notation that uses a rule to describe the members of the set. **Ex:** $\{x \mid x > 5\}$ *"x such that x is greater than 5"*
Subset	A set of elements that are members of a larger set **Ex.** $A \subset B$ *"set A is a subset of set B"*
Equal Sets	Two or more sets having exactly the same elements
Intersection of Sets	The set of elements common to both sets A AND B →Think overlapping sets **Notation:** $A \cap B$ "A intersect B"
Union of Sets	The set of elements that are members of set A OR of set B OR of both sets →Think all of set A with all of set B **Notation:** $A \cup B$ "A union B"

Important Symbols					
$\in$	***"element of"***	$\subset$	***"subset of"***	$\varnothing$	***"null set"***
|	***"such that"***	$\cap$	***"intersection"***	$\cup$	***"union"***

Examples:

1. List all possible subsets of $A = \{1, 2, 3\}$

{1,2,3}   {1,2}   {1,3}   {2,3}

{1}   {2}   {3} $\varnothing$

2. Determine whether each of the following is true or false.

a. **F** $2 \notin \{1, 2, 3\}$

b. **T** $\{y, x, z\} \subset \{x, y, z\}$

c. **T** $\{y, x, z\} = \{x, y, z\}$

d. **F** $\varnothing \in \{x, y, z\}$ The null set is a *subset* of every set, not an *element*.

e. **F** $\{w, x, y, z\} \subset \{x, y, z\}$

3. Write each of the following sets using *set builder notation.*

a. {2, 4, 6, 8, 10} **{*x* | *x* an even integer between 2 and 10 inclusive}**

b. {1, 2, 3, 4, 5} **{*x* | *x* an integer between 1 and 5 inclusive}**

4. Write the following set using the *listing method*.

a. {*x* | *x* is a letter in the word ALGEBRA} **{A, L, G, E, B, R}**

b. {*x* | *x* is an even integer between 6 and 12 inclusive} **{6, 8, 10, 12}**

5. Given set *A*= $\{1, 2, 3, 4, 5, 6\}$ and set *B*= $\{2, 4, 6, 8\}$, find the following:

a. $A \cap B$ {2, 4, 6}

b. $A \cup B$ {1, 2, 3, 4, 5, 6, 8}

Natural Numbers	The set of positive counting numbers $\{1, 2, 3, \ldots\}$
Whole Numbers	The set of natural numbers AND zero $\{0, 1, 2, 3, \ldots\}$
Integers	The set of whole numbers AND their opposites $\{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$
Rational Numbers	The set of numbers than can be written as a quotient of two integers. Rational means fractional (decimals that terminate or repeat) **Ex.** $-\frac{5}{3}, \frac{7}{9}, 0.5, \frac{4}{1}, 0.\overline{33}$
Irrational Numbers	The set of numbers that are not rational (13 and nonterminating decimal numbers) **Ex.** $\sqrt{7}, \pi, -\sqrt{13}$
Real Numbers	The set of rational numbers and irrational numbers

Examples:

6. Given set $B = \left\{-2, \frac{1}{3}, \sqrt{2}, 0, \frac{8}{3}, \pi, \sqrt{36}\right\}$, list the following:

a. Rational Numbers $\left\{-2, \frac{1}{3}, 0, \frac{8}{3}, \sqrt{36}\right\}$

b. Irrational Numbers $\left\{\sqrt{2}, \pi\right\}$

c. Integers $\left\{-2, 0, \sqrt{36}\right\}$

d. Whole Numbers $\left\{0, \sqrt{36}\right\}$

e. Natural Numbers $\left\{\sqrt{36}\right\}$

Properties of Fractions	
$\frac{ka}{kb}=\frac{a}{b}$	In order to reduce a fraction, cancel common factors
$\frac{a}{b}+\frac{c}{b}=\frac{a+c}{b}$ $\quad$ $\frac{a}{b}-\frac{c}{b}=\frac{a-c}{b}$	In order to add or subtract fractions, the denominators must be the same →find the least common denominator (LCD)
$\frac{a}{b}+\frac{c}{d}=\frac{a}{b}\bullet\frac{d}{d}+\frac{c}{d}\bullet\frac{b}{b}=\frac{ad+cb}{bd}$	
$\frac{a}{b}\bullet\frac{c}{d}=\frac{ac}{bd}$	In order to multiply fractions, multiply numerators together and multiply denominators together →multiplication goes straight across
$\frac{a}{b}\div\frac{c}{d}=\frac{a}{b}\bullet\frac{d}{c}$	In order to divide fractions, multiply by the reciprocal →division flips the second fraction then changes to multiplication

Properties of Negatives	
$-(-a)=a$ $(-a)(-b)=ab$ $\frac{-a}{-b}=-\frac{-a}{b}=-\frac{a}{-b}=\frac{a}{b}$	An even number of negatives results in a positive number
$(-a)b=-ab$ $-1(a)=-a$ $\frac{-a}{b}=-\frac{a}{b}=\frac{a}{-b}$	An odd number of negatives results in a negative number

Examples:

1. $\frac{3}{2}+\frac{-2}{3}=\frac{5}{6}$

2. $\frac{3}{2}\div-\frac{2}{3}=-\frac{9}{4}$

3. $\frac{3}{2}\bullet\frac{2}{-3}=-1$

Properties of Equality		
Name	**Example**	**Translation**
Reflexive Property	$a = a$	Every number is equal to itself.
Symmetric Property	If $a = b$, then $b = a$.	The order in which you write an equation is insignificant.
Transitive Property	If $a = b$ and $b = c$, then $a = c$.	Two numbers equal to the same number are equal to each other.
Addition Property	If $a = b$, then $a + c = b + c$ and $c + a = c + b$.	If you add to one side of an equation, then you must add the same amount to the other. → Equations must be balanced!
Multiplication Property	If $a = b$, then $ac = bc$ and $ca = cb$.	If you multiply on one side of an equation, you must multiply the same amount on the other. → Equations must be balanced!
Distributive Property	$a(b + c) = ab + ac$ $(b + c)a = ba + ca$	In order to multiply a number by a quantity, you must multiply that number by each number inside the quantity

Zero Properties	
$a \bullet 0 = 0 \bullet a = 0$	Any number multiplied by zero is zero
If $\boldsymbol{AB = 0}$, then $\boldsymbol{A = 0}$ or $\boldsymbol{B = 0}$, or $\boldsymbol{A = 0}$ and $\boldsymbol{B = 0}$	If the **product of two numbers is zero**, then **one or both of the numbers must be zero**.
Example: If $\boldsymbol{(x+2)(x-3)=0}$, then $\boldsymbol{x+2=0}$ or $\boldsymbol{x-3=0}$.	

Field Properties of Real Numbers		
Name	**Example**	**Translation**
Commutative Property	$a + b = b + a$ $ab = ba$	An operation is commutative when a change in order yields the same result
Associative Property	$(a + b) + c = a + (b + c)$ $(ab)c=a(bc)$	An operation is associative when a change in grouping yields the same result
Identity Property for Addition	$a + 0 = a$ and $0 + a = a$	Zero is the *additive identity* Adding zero to any number will not change the number
Identity Property for Multiplication	$a \cdot 1 = a$ and $1 \cdot a = a$	One is the *multiplicative identity* Multiplying by one will not change the number
Inverse Property of Addition – Property of Opposites	$a + (-a) = 0$ and $(-a) + a = 0$	Adding a number and its opposite will result in zero
Inverse Property of Multiplication - Property of Reciprocals	$a \cdot \frac{1}{a} = 1$ and $\frac{1}{a} \cdot a = 1$	Multiplying a number and its reciprocal will result in one

Examples:

1. Write the property that justifies each step.

$(7+3)+[(-7)+(-3)]$	$=[(7+3)+(-7)]+(-3)$	**Associative Property Of Addition**
	$=[7+(-7)+3]+(-3)$	**Associative and Commutative Properties**
	$=[0+3]+(-3)$	**Inverse Property of Addition (Property of Opposites)**
	$=3+(-3)$	**Identity Property of Addition**
	$=0$	**Inverse Property of Addition (Property of Opposites)**

2. Write the property that justifies each step.

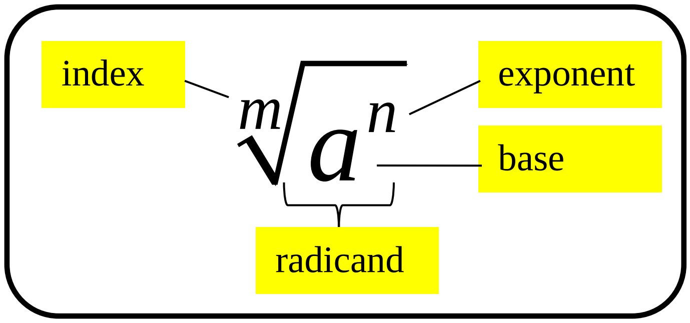
$3-\{4-2[3+2+(-3)]\}$	$=3-\{4-2[2+3+(-3)]\}$	**Commutative Property Of Addition**
	$=3-\{4-2[2+0]\}$	**Inverse Property of Addition (Property of Opposites)**
	$=3-\{4-2[2]\}$	**Identity Property of Addition**
	$=3-\{4-4\}$	**Simplify**
	$=3-\{0\}$	**Inverse Property of Addition (Property of Opposites)**
	$=3$	**Identity Property of Addition**

1.2 Radicals

Essential Question(s):

- **How do you simplify radicals?**
- **How do you convert radicals to rational exponents and vice versa?**

Anatomy of a radical:



Simplifying Radicals	
$\sqrt[n]{a^n} = \left(\sqrt[n]{a}\right)^n = a$	When the index and the exponent are equal, they cancel **Note:** When *n* is even, the result should include the absolute value $\sqrt{x^2} = \lvert x \rvert$
$\sqrt[3]{x^4} = x\sqrt[3]{x}$	Exponents in the radicand must be less than the index
$\sqrt[6]{x^3} = \sqrt{x}$	Exponents in the radicand CANNOT have a common factor other than one with the index
$\frac{1}{\sqrt{9}} = \frac{1}{3}$	No radicals in the denominator
$\sqrt{\frac{1}{4}} = \frac{\sqrt{1}}{\sqrt{4}} = \frac{1}{2}$	No fractions within the radical

Examples:

1. $\sqrt{12} = 2\sqrt{3}$

2. $\sqrt{32} = 4\sqrt{2}$

3. $\sqrt[3]{8} = 2$

4. $\sqrt[3]{625} = 5\sqrt[3]{5}$

5. $\sqrt[4]{x^4} = |x|$

6. $\sqrt[5]{x^5} = x$

7. $\sqrt[4]{x^5} = |x|\sqrt[4]{x}$

8. $\sqrt[3]{x^5} = x\sqrt[3]{x^2}$

9. $\sqrt[4]{x^6} = \sqrt{x^3} = |x|\sqrt{x}$

10. $\sqrt[10]{x^5} = \sqrt{x}$

11. $\sqrt{18x^5y^2z^3} = 3x^2|yz|\sqrt{2xz}$

12. $\sqrt[3]{8x^5y^3} = 2xy\sqrt[3]{x}$

Properties of Radicals:		
Name	**Formula**	**Translation**
Product Property	$a\sqrt{x} \cdot b\sqrt{y} = ab\sqrt{xy}$	**Coefficients multiply together** and **radicands multiply together** →Outside times outside Inside times inside ➢ Make sure to fully the simplify the resulting radical!
Division Property	$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$	**The square root of a quotient** is **the quotient of the square roots** → You can break apart division
Nested Radical	$\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a}$	**To take the radical of a radical, multiply the indexes then simplify**

Examples (Assume all variables are positive- no need to include absolute values):

1. $2\sqrt{6} \cdot \sqrt{2} = 2\sqrt{12} = 4\sqrt{3}$

2. $3\sqrt[3]{4} \cdot \sqrt[3]{10} = 3\sqrt[3]{40} = 6\sqrt[3]{5}$

3. $\sqrt[3]{\frac{x^2}{8}} = \frac{\sqrt[3]{x^2}}{2}$

4. $\sqrt[4]{\frac{x}{16}} = \frac{\sqrt[4]{x}}{2}$

5. $\sqrt[4]{27a^3b^3} \cdot \sqrt[4]{3a^5b^3}$

$= \sqrt[4]{81a^8b^6} = 3a^2b\sqrt[4]{b^2} = 3a^2b\sqrt{b}$

6. $\sqrt[3]{12x^3y^2z^7} \cdot \sqrt[3]{3xy^2z}$

$= \sqrt[3]{36x^4y^4z^8} = xyz^2\sqrt[3]{36xyz^2}$

7. $\sqrt[3]{\sqrt[4]{x}} = \sqrt[12]{x}$

8. $\sqrt{\sqrt{x^4}} = \sqrt[4]{x^4} = x$

Properties of Radicals:		
Name	**Example**	**Translation**
Addition Property	$\sqrt{a} + \sqrt{a} = 2\sqrt{a}$ $4\sqrt{a} - \sqrt{a} = 3\sqrt{a}$	**When the radicand and the index are equal, add/subtract the coefficients** →You must have like terms to add or subtract

Examples:

1. $6\sqrt{2} + 2\sqrt{2} = 8\sqrt{2}$

2. $4\sqrt[3]{5} - \sqrt[3]{5} = 3\sqrt[3]{5}$

3. $3\sqrt[5]{2x^2y^3} - 8\sqrt[5]{2x^2y^3} = -5\sqrt[5]{2x^2y^3}$

4. $5\sqrt[3]{mn^2} - 3\sqrt{mn} - 2\sqrt[3]{mn^2} + 7\sqrt{mn}$

$= 3\sqrt[3]{mn^2} + 4\sqrt{mn}$

Rationalizing the Denominator

Rationalizing the denominator is a process of removing a radical from the denominator of a fraction.

Example: $\dfrac{4}{\sqrt{3}}$ Step 1: $\dfrac{4}{\sqrt{3}} \cdot \dfrac{\sqrt{3}}{\sqrt{3}}$ Step 2: $\dfrac{4\sqrt{3}}{\sqrt{9}}$ Step 3: $\dfrac{4\sqrt{3}}{3}$

Form of the denominator:	Multiply numerator AND denominator by:
$\sqrt{b}$	$\sqrt{b}$ *multiply by the **same** square root*
$\sqrt[n]{b}$	$\sqrt[n]{b^?}$ *multiply to get an **exponent of n***
$a+\sqrt{b}$	$a-\sqrt{b}$ *multiply by the conjugate*
$a-\sqrt{b}$	$a+\sqrt{b}$ *multiply by the conjugate*

Examples:

1. $\dfrac{6}{\sqrt{2x}} \cdot \dfrac{\sqrt{2x}}{\sqrt{2x}} = \dfrac{6\sqrt{2x}}{2x} = \dfrac{3\sqrt{2x}}{x}$

2. $\sqrt{\dfrac{x^2}{y^3}} = \dfrac{x}{y\sqrt{y}} \cdot \dfrac{\sqrt{y}}{\sqrt{y}} = \dfrac{x}{y^2}$

3. $\dfrac{10x^3}{\sqrt[3]{4x}} \cdot \dfrac{\sqrt[3]{2x^2}}{\sqrt[3]{2x^2}} = \dfrac{10x^3\sqrt[3]{2x^2}}{2x} = 5x^2\sqrt[3]{2x^2}$

4. $\sqrt[3]{\dfrac{8c}{9d^5}} = \dfrac{\sqrt[3]{8c}}{d\sqrt[3]{3^2d^2}} \cdot \dfrac{\sqrt[3]{3d}}{\sqrt[3]{3d}} = \dfrac{\sqrt[3]{24cd}}{3d^2}$

5. $\dfrac{\sqrt{x}+2}{2\sqrt{x}+3} \cdot \dfrac{2\sqrt{x}-3}{2\sqrt{x}-3} = \dfrac{2\sqrt{x^2}-3\sqrt{x}+4\sqrt{x}-6}{4\sqrt{x^2}-9} = \dfrac{2x+\sqrt{x}-6}{4x-9}$

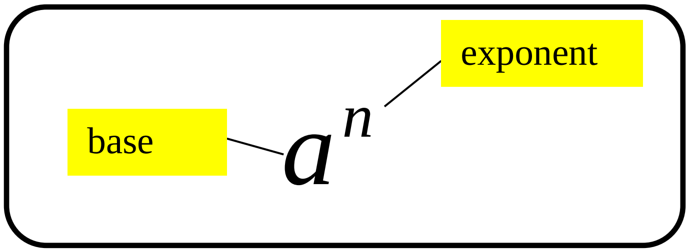
6. $\dfrac{3+\sqrt{5}}{3-\sqrt{5}} \cdot \dfrac{3+\sqrt{5}}{3+\sqrt{5}} = \dfrac{9+2\sqrt{5}+5}{9-5} = \dfrac{14+2\sqrt{5}}{4} = \dfrac{7+\sqrt{2}}{2}$

1.3 Exponents

Essential Question(s):

- **How do you simplify exponential expressions?**
- **How do you express numbers using scientific notation?**

Anatomy of an exponent:



Integer Exponents	
For *n* a positive integer, $a^n = a \cdot a \cdot .... \cdot a \to n$ factors of a	*"a"* multiplies by itself *"n"* times
For *n* = 0 $a^0 = 1 \qquad a \neq 0$	Anything to the zero power is ONE Caution: 0^0 is not defined
For *n* a negative integer: $a^{-n} = \frac{1}{a^n}$	A negative exponent results in a reciprocal →flip the fraction! Caution: A negative exponent DOES NOT result in a negative number!!

Examples:

1. $636^0 = 1$

2. $\frac{1}{10^{-3}} = 10^3 = 1000$

3. $\frac{u^{-7}}{v^{-3}} = \frac{v^3}{u^7}$

4. $(x^2)^0 = x^0 = 1$

5. $10^{-5} = \frac{1}{10^5} = \frac{1}{10000} = 0.00001$

6. $\frac{1}{x^{-4}} = x^4$

Properties of Integer Exponents	
$a^m a^n = a^{m+n}$	If bases are the same, add exponents
$\left(a^n\right)^m = a^{mn}$	If bases are the same, multiply exponents
$\frac{a^m}{a^n} = \begin{cases} a^{m-n} \\ \frac{1}{a^{n-m}} \end{cases} \qquad a \neq 0$	If bases are the same, subtract exponents
$(ab)^m = a^m b^m$	If bases are different, distribute exponents
$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m} \qquad b \neq 0$	If bases are different, distribute exponents

Examples:

1. $(5x^{-3})(3x^4) = 15x$

2. $\frac{9y^{-7}}{6y^{-4}} = \frac{3}{2y^3}$

3. $\left(\frac{x^2}{y^4}\right)^{-3} = \frac{y^{12}}{x^6}$

4. $(3x^4y^{-3})^{-2} = 3^{-2}x^{-8}y^6 = \frac{y^6}{9x^8}$

5. $\frac{1}{(a-b)^{-2}} = (a-b)^2 = a^2 - 2ab + b^2$

6. $(2y^2)(3y^5) = 6y^7$

7. $\frac{6m^{-2}n^3}{15m^{-1}n^{-2}} = \frac{2n^5}{5m}$

8. $\left(\frac{x^{-3}}{y^4y^{-4}}\right)^{-3} = x^9$

9. $2x^4 - (-2x)^4 = 2x^4 - 16x^4 = -14x^4$

10. $(x+y)^{-2} = \frac{1}{(x+y)^2} = \frac{1}{x^2 + 2xy + y^2}$

Anatomy of a rational exponent:

$a^{\frac{n}{m}}$

base: a; exponent: n; index: m

Anatomy of a radical:

$\sqrt[m]{a^n}$

index: m; exponent: n; base: a

Examples:

1. Change from rational exponent form to radical form.

a. $u^{\frac{1}{5}} = \sqrt[5]{u}$

b. $w^{\frac{2}{3}} = \sqrt[3]{u^2}$

c. $(3xy)^{-\frac{3}{5}} = \sqrt[5]{(3xy)^{-3}} = \dfrac{1}{\sqrt[5]{27x^3y^3}}$

d. $ab^{\frac{-2}{3}} = \dfrac{a}{\sqrt[3]{b^2}}$

2. Change from radical form to rational exponent form.

a. $\sqrt[4]{9u} = (9u)^{\frac{1}{4}}$

b. $\sqrt[3]{x^3 + y^3} = (x^3 + y^3)^{\frac{1}{3}}$

c. $\sqrt[7]{(-2x)^4} = (-2x)^{\frac{4}{7}}$

d. $-\sqrt[7]{(2x)^4} = -(2x)^{\frac{4}{7}}$

Evaluate:

1. $4^{\frac{1}{2}} = \sqrt{4} = 2$

2. $-4^{\frac{1}{2}} = -\sqrt{4} = -2$

3. $(-4)^{\frac{1}{2}} = \sqrt{-4} = \text{not real}$

4. $8^{\frac{1}{3}} = \sqrt[3]{8} = 2$

5. $(-8)^{\frac{1}{3}} = \sqrt[3]{-8} = -2$

6. $0^{\frac{1}{8}} = \sqrt[8]{0} = 0$

7. $9^{\frac{3}{2}} = \sqrt{9^3} = 9\sqrt{9} = 9(3) = 27$

8. $(-27)^{\frac{4}{3}}$

$= \sqrt[3]{(-27)^4} = -27\sqrt[3]{-27} = -27(-3) = 81$

Simplify. Write your answer in *exponential form* (using positive exponents only) and *simplest radical form (where applicable)*.

1. $\left(2x^{-\frac{3}{4}}y^{\frac{1}{4}}\right)^4 = 16x^{-3}y = \dfrac{16y}{x^3}$

2. $\left(\dfrac{4x^{-2}}{y^4}\right)^{-\frac{1}{2}} = \dfrac{2^{-1}x}{y^{-2}} = \dfrac{xy^2}{2}$

Exp. Form

3. $\left(3x^{\frac{1}{3}}\right)\left(2x^{\frac{1}{2}}\right) = 6x^{\frac{2}{6}+\frac{3}{6}} = 6x^{\frac{5}{6}} = 6\sqrt[6]{x^5}$

Rad. Form

Exp. Form

4. $\left(5y^{\frac{3}{4}}\right)\left(2y^{\frac{1}{3}}\right) = 10y^{\frac{9}{12}+\frac{4}{12}} = 10y^{\frac{13}{12}} = 10\sqrt[12]{y^{13}} = 10y\sqrt[12]{y}$

Rad. Form