## CHAPTER 11 - MEASUREMENTS of Figures and Solids

| Section: | $\mathbf{1 1 - 6} \quad$ Volume of Prisms and Pyramids |
| :--- | :--- |
| Essential <br> Question | How does the volume of a prism relate to the volume of a pyramid? |

## Warm Up:

$\square$
Key Vocab:

| Volume | The number of cubic units contained in the interior of a solid. |
| :--- | :--- |
| Polyhedron | A solid that is bounded by <br> polygons, called faces that enclose <br> a single region of space. <br> Plural: polyhedra or polyhedrons. |
| Prism | A polyhedron with two bases that <br> are congruent polygons in parallel <br> planes |
| Pyramid | A polyhedron in which the base is a <br> polygon and the lateral faces are <br> triangles with a common vertex, <br> called the vertex of the pyramid. |


| Platonic Solid |  |  |  |
| :--- | :--- | :--- | :---: |
| Five regular polyhedra |  |  |  |
| Named after the Greek mathematician and philosopher Plato. |  |  |  |

## Postulates:

| Volume Addition Postulate |
| :---: |
| The volume of a solid is the sum of the volumes of all its nonoverlapping parts. |
| The sum of the parts equals the whole |

## Formulas:

| Volume of a Prism |  |
| :---: | :---: |
| $V=B h$ <br> where $B$ is the area of the base and $h$ is the height. |  |


| Volume of a Pyramid |  |  |  |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\qquad V=\frac{1}{3} B h$ |  |  |  |  |  |  |  |
| where $B$ is the area of the base and $h$ is the |  |  |  |  |  |  |  |
| height. |  |  |  |  |  |  |  |

## Show:

Ex 1: The volume of the gift box cube $108 \mathrm{in}^{3}$. Find the value of $x$.

$$
\begin{aligned}
108 & =x^{3} \\
\sqrt[3]{108} & =x \\
4.76 \text { in } & \approx x
\end{aligned}
$$



Ex 2: Find the volume of the right hexagonal prism.


$$
\begin{aligned}
& B=\frac{1}{2}(6 \sqrt{3})(72)=216 \sqrt{3} \\
& V=216 \sqrt{3}(7) \approx 2618.86 \mathrm{~cm}^{3}
\end{aligned}
$$

Ex 3: Find the volume of the square pyramid.

$$
V=\frac{1}{3}(14)^{2}(10) \approx 653.33 \mathrm{~m}^{3}
$$



Ex 5: Find the volume of the triangular pyramid.


$$
V=\frac{1}{3}\left(\frac{1}{2} \cdot 3 \cdot 4\right) 3=6 \mathrm{in}^{3}
$$

Ex 4: The Pyramid of the Sun in Teotihuacan, Mexico, is a regular square pyramid with height 63 m and volume $970,725 \mathrm{~m}^{3}$. Find the side length of the base.

$$
\begin{aligned}
970,725 & =\frac{1}{3} 63 x^{2} \\
46,225 & =x^{2} \\
215 \mathrm{~m} & =x
\end{aligned}
$$

| Section: | $\mathbf{1 1 - 7} \quad$ Volume of Cylinders and Cones |
| :--- | :--- |
| Essential <br> Question | How does the volume of a cylinder relate to the volume of a cone? |

Warm Up:
$\square$
Key Vocab:

| Cylinder | A solid that has two circular bases. |  |
| :--- | :--- | :--- |
| Cone | A solid that has one circular base <br> and a vertex that is not in the same <br> plane as the base. |  |

Formulas:

$$
V=B h=\pi r^{2} h
$$

where $B$ is the area of the base, $h$ is the height, and $r$ is the radius of the base.


| Volume of a Cone |  |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
| $\qquad V=\frac{1}{3} B h=\frac{1}{3} \pi r^{2} h$ |  |  |  |  |  |
| where $B$ is the area of the base, $h$ is the |  |  |  |  |  |
| height, and $r$ is the radius of the base. |  |  |  |  |  |

## Show:

Ex 1: Find the volume of each solid. (Round answers to the nearest hundredths)
a.)
b.) Find the volume of the skew cone.


$$
V=\frac{1}{3} \pi(1.1)^{2}(2.7) \approx 3.42 \mathrm{~cm}^{3}
$$

Ex 3: Find the volume of the right cone. (Round your answer to two decimal places)


$$
\begin{array}{ll}
\tan (70)=\frac{25}{r} & V=\frac{1}{3} \pi\left(\frac{25}{\tan (70)}\right)^{2} \\
r=\frac{25}{\tan (70)} & V \approx 2167.61 \mathrm{~cm}^{3}
\end{array}
$$

Ex 4: Find the volume of the solid shown which is formed by two cones. (Round your answer to two decimal places)


$$
\begin{aligned}
& V=\frac{1}{3} \pi(10)^{2}(16)+\frac{1}{3} \pi(10)^{2}(18) \\
& V \approx 3560.47 \mathrm{~mm}^{3}
\end{aligned}
$$

Ex 5: You are making coffee using a cone-shaped filter. It takes 14 minutes to brew coffee. Find the flow rate of the coffee in cubic inches per minute. (Round your answer to two decimal places)


$$
\begin{aligned}
& V=\frac{1}{3} \pi(3)^{2}(3.5)=10.5 \pi \mathrm{in}^{3} \\
& \text { flow rate }=\frac{10.5 \pi}{14} \approx 2.36 \mathrm{in}^{3} / \mathrm{min}
\end{aligned}
$$

1) b.)

2) 

$$
B=\pi(40)^{2}=1600 \pi
$$

3) $\quad V=1600 \pi(10) \approx 50265.48 \mathrm{ft}^{3}$

Ex 4: Find the volume of the oblique cylinder when $h=12 \mathrm{~m}$ and $r=9 \mathrm{~m}$.


$$
V=9^{2} \pi(12) \approx 3053.6 \mathrm{~m}^{3}
$$

Ex 5: A cistern is a large tank used to collect rainwater. It is made of concrete that is 3 inches thick and is open at the top. Find the volume of the concrete needed to make the sides and bottom of the cistern. (Round you answer to the nearest hundredths)


$$
\begin{aligned}
V_{\text {total }} & =\pi(18)^{2}(20)=6480 \pi \\
V_{\text {inner }} & =\pi(15)^{2}(17)=3825 \pi \\
& \begin{aligned}
V_{\text {outer }} & =V_{\text {total }}-V_{\text {inner }} \\
& =6480 \pi-3825 \pi \\
& =2655 \pi \approx 8340.93 \mathrm{in}^{3}
\end{aligned}
\end{aligned}
$$

| Section: | $\mathbf{1 1 - 8} \quad$ Surface Area and Volume of Spheres |
| :--- | :--- |
| Essential <br> Question | How do you find the volume of a sphere? |

Warm Up:


## Key Vocab:

| Sphere | The set of all points in space <br> equidistant from a given point <br> called the center of the sphere. |
| :--- | :--- |
| Great Circle | The intersection of a sphere and a plane that contains the center of the <br> sphere. |
| Hemisphere | Half of a sphere formed when a great circle separates a sphere into two <br> congruent halves. |

Surface Area of a Sphere- The surface
area $S$ of a sphere is $S=4 \pi r^{2}$, where $r$ is the
radius of the sphere.
Volume of a Sphere- The volume $V$ of a
sphere is $V=\frac{4}{3} \pi r^{3}$, where $r$ is the radius of
the sphere.

## Show:

Ex 1: Find the surface area of the sphere with a radius of 12 ft .

$$
S A=4 \pi(12)^{2}=576 \pi \mathrm{ft}^{2} \approx 1809.56 \mathrm{ft}^{2}
$$

Ex 2: The surface area of a sphere is $40.96 \pi \mathrm{in}^{2}$. What is the diameter of the sphere?
A. 3.2 in
B. 8.6 in
C. 6.4 in
D. 40.96 in

Ex 3: A globe of Earth is a model of a sphere. The circumference of this globe is $18 \pi$ in. Find the surface area of the globe.


$$
\begin{aligned}
18 \pi & =2 \pi r \\
9 & =r
\end{aligned}
$$

$$
S A=4 \pi(9)^{2} \approx 1017.88 \mathrm{in}^{2}
$$

Ex 4: This beach ball has a diameter of 15 inches. Find its volume.


$$
\begin{aligned}
2 r & =15 \\
r & =7.5
\end{aligned}
$$

$$
V=\frac{4}{3} \pi(7.5)^{3} \approx 1767.15 \mathrm{in}^{3}
$$

Ex 5: Find the volume of the composite solid.


$$
\begin{aligned}
& V=V_{\text {cylinder }}+V_{\text {sphere }} \\
& V=\pi(2)^{2}(7)+\frac{4}{3} \pi(2)^{3} \\
& V=28 \pi+\frac{32}{3} \pi \approx 121.47 \mathrm{~cm}^{3}
\end{aligned}
$$

