NAME ______________________ DATE ______________

6–3 Indirect Proof

Objective: Write indirect proofs in paragraph form.

Most of the proofs you have seen or written have been **direct proofs.** You reasoned directly from the given to the conclusion, using definitions, theorems, and postulates. Sometimes it is difficult or even impossible to find a direct proof. In that case, it may be possible to write an **indirect proof.**

Indirect reasoning is used in everyday life. Consider the following example, in which you are trying to convince your neighbor.

> You say that my dog, Rex, dug a hole in your yard on July 15. Temporarily assume that Rex did dig the hole. Then he would have been in your yard on July 15. But I have bills that show Rex was in the veterinarian's kennel from July 14 to July 17. Rex couldn't have been in two places at once. Therefore, Rex is not the dog that dug the hole in your yard.

In an indirect proof, you begin by assuming temporarily that what you want to prove is not true. In the example above, you assumed that your dog dug the hole.

Example 1	*Solution*
Write the first sentence of an indirect proof of each conditional shown.	
a. If $AB = BC$, then $\triangle ABC$ is not scalene.	a. Assume temporarily that $\triangle ABC$ is scalene.
b. If $n^2 > 6n$, then $n \neq 4$.	b. Assume temporarily that $n = 4$.
c. If $m\angle 1 = m\angle 2$, then $\overline{XY} \parallel \overline{CD}$.	c. Assume temporarily that $\overline{XY} \nparallel \overline{CD}$.

Write the first sentence of an indirect proof of each conditional shown.

1. If the sap of a plant is milky, then the plant is poisonous.
2. If $\overline{XY} \parallel \overline{CD}$, then $m\angle 1 = m\angle 2$.
3. If $\overline{AB} \nparallel \overline{CD}$, then $ABCD$ is not a parallelogram.
4. If M is the midpoint of $\overline{AB}$, then $AM = MB$.
5. If $AC \neq BD$, then $ABCD$ is not a rectangle.

An indirect proof is usually written in paragraph form. After making the temporary assumption, you reason logically until you reach a contradiction of a known fact.

Indirect proofs are often used to prove a conclusion that is a negation, such as not equal or not parallel.

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6–3 Indirect Proof *(continued)*

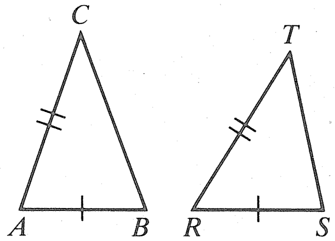
How to Write an Indirect Proof

1. Assume temporarily that the conclusion is not true.
2. Reason logically until you reach a contradiction of a known fact.
3. Point out that the temporary assumption must be false, and that the conclusion must then be true.

Example 2

Given: $AC = RT$; $AB = RS$; $\angle A \not\cong \angle R$

Prove: $BC \neq ST$



Solution

Assume temporarily that $BC = ST$. Then $\triangle ABC \cong \triangle RST$ by SSS, and $\angle A \cong \angle R$ since corr. parts of $\cong$ $\triangle$s are $\cong$. But this contradicts the given information that $\angle A \not\cong \angle R$. Therefore the temporary assumption that $BC = ST$ must be false. It follows that $BC \neq ST$.

Number the sentences in an order that completes an indirect proof.

6. Given: $\triangle ABC$; $\overline{AB} \cong \overline{BC}$
   Prove: $m\angle A \neq 90$

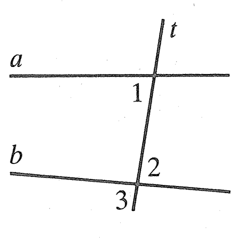
   (___) Then $m\angle C = 90$, and $m\angle A + m\angle B + m\angle C = 90 + m\angle B + 90 > 180$.
   (___) But this contradicts the fact that the sum of the measures of the angles of a triangle is 180.
   (___) Assume temporarily that $m\angle A = 90$.
   (___) It follows that $m\angle A \neq 90$.
   (___) Therefore the temporary assumption that $m\angle A = 90$ must be false.

7. If $n^2 > 6n$, then $n \neq 4$.

   (___) Therefore the temporary assumption that $n = 4$ must be false.
   (___) Then $n^2 = 16$ and $6n = 24$.
   (___) It follows that $n \neq 4$.
   (___) Assume temporarily that $n = 4$.
   (___) But this contradicts the given fact that $n^2 > 6n$, since $16 \not> 24$.

Write an indirect proof in paragraph form.

8. Given: Transversal t cuts lines a and b; $m\angle 1 \neq m\angle 2$
   Prove: $m\angle 1 \neq m\angle 3$



9. Given: Scalene $\triangle REN$
   Prove: $\angle R \not\cong \angle N$